Quantile VARs and Macroeconomic Risk Forecasting

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Recent rises in macroeconomic volatility has prompted the introduction of quantile VAR (QVAR) models for modeling risk. We evaluate its ability to forecast macroeconomic risks in a pseudo-out-of-sample experiment spanning 112 US monthly variables over a period of 40 years for horizons of 1 to 12 months by comparing with three parametric benchmarks: a Gaussian VAR (VAR-N), a VAR-GARCH and a VAR with stochastic volatility (VAR-SV). QVAR statistically significantly improves over VAR-N for half of our variables for forecasting quantiles and density in the tails. Gains can be large, ranging between 10 and 30%. Much of these gains are concentrated in the labor market at all horizons, as well as interest and exchange rates at shorter horizons. We also find QVAR provides a robust approximation to conditional distributions: it often beats, but almost never does significantly worse than any of the parametric alternatives. Finally, augmenting the QVAR model with factors (QFAVAR) estimated by principal components or the quantile factors recently introduced by Chen, Dolado, and Gonzalo (2021) significantly improves macroeconomic risk forecasting only in a handful of cases, most of them in the labor market. In general, QVAR and QFAVAR perform equally well. We conclude that both are adequate tools for modeling macroeconomic risks.

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1. Introduction

The rise in macroeconomic volatility experienced during the 2007 financial crisis and the COVID pandemic put an end to the Great Moderation and increased the interest in modeling macroeconomic risk. Work by Giglio, Kelly, and Pruitt (2016) and Adrian, Boyarchenko, and Giannone (2019) popularized the use of quantile regressions in this context, finding evidence that financial stress leads to asymmetry in output growth. Many studies applied those methods in a single equation framework focusing on the predictive power of financial indicators for risk to output growth (e.g., Figueres and Jarociński (2020), Adams et al. (2021) and Iseringhausen (2021)) and inflation (e.g., Manzan and Zerom (2013), Manzan (2015) and López-Salido and Loria (2020)). Others proposed using quantile regressions as part of a structural analysis studying the effects of shocks on the conditional distribution of output growth (Loria, Matthes, and Zhang 2023) or to distinguish between shocks to upside, downside and total uncertainty (Forni, Gambetti, and Sala 2021). Quantile regression methods were also extended to a VAR setting by White, Kim, and Manganelli (2015), Chavleishvili and Manganelli (2021), Chavleishvili et al. (2021) and Ruzicka (2021) for forecasting, scenario analysis, macroprudential risk management and quantile impulse responses.

The use of linear quantile regression models is primarily motivated by an appeal to their robustness as approximations to conditional quantiles and distributions. Economic theory can justify a wide variety of VAR processes for modeling conditional distributions[1], but all of them requires committing to a particular functional form. Since linear quantile regressions provide a weighted least square optimal linear approximation to the true conditional quantiles (Angrist, Chernozhukov, and Fernández-Val 2006), they have been employed to produce forecasts or insights regarding macroeconomic risks in ways which are hopefully robust to the unknown form of the underlying data generating process.

In such a context, we can understand the popularity of quantile regression methods for studying and forecasting macroeconomic risk. Several researchers have recently proposed a quantile VAR (QVAR) model (White, Kim, and Manganelli (2015), Chavleishvili

[1]Occasionally binding collateral constraints (Aiyagari and Gertler 1999) or a kinked Phillips curve (Benigno and Eggertsson 2023) suggests using a threshold VAR. The model in Acemoglu and Scott (1997) imply a smooth transition process for output where the transition function emerges from firm heterogeneity as only some firms opt to invest at a given point in time. Real options arguments (Bernanke (1983) and McDonald and Siegel (1986)) and frictions to the supply of credit (e.g., (Adrian and Boyarchenko 2012) and (Brunnermeier and Sannikov 2014)) can motivate the use of volatility-in-means effects (e.g., Elder and Serletis (2010)).

and Manganelli (2021), Ruzicka (2021) and Montes-Rojas (2021)), but its forecasting performance has yet to be assessed.

The first contribution of this paper to provide an extensive evaluation of the predictive performance of the QVAR model. Other papers explored a similar comparison in a single equation setting between quantile regression models and AR-GARCH models (e.g., Brownlees and Souza (2021), Iseringhausen (2021) and Kipriyanov (2022)). Other compared quantile regression models with more sophisticated parametric VAR alternatives (e.g., Carriero, Clark, and Marcellino (2021) and Caldara et al. (2021)), but the QVAR model has yet to be compared to parametric alternatives. Throughout this paper, we target conditional densities with a focus on one or both tails of conditional distributions, as well as the 5th and 95th quantiles. The comparison features 112 US monthly macroeconomic variables and an out-of-sample period of over 40 years with forecasting horizons of between a month and a year.

This is in stark contrast with the typical forecasting evaluation in this literature which focuses on a few targets. This exercise is built around bivariate VAR models where the target variable is paired with the National Financial Conditions Index (NFCI). This is perhaps the most interesting comparison as it is the most commonly used predictor in the growth-at-risk literature following Adrian, Boyarchenko, and Giannone (2019). We also know credit shocks to be important drivers of macroeconomic fluctuations for a large number of variables (Boivin, Giannoni, and Stevanović 2020) so financial stress may be relevant to many of our target variables, insofar as it captures this type of shock. We compare QVAR on this basis with three parametric alternatives. The first alternative is a Gaussian VAR (VAR-N) which allows us to evaluate when and how much gain there is to moving beyond iid disturbances. We also include a VAR-GARCH model as in Normandin and Phaneuf (2004), Bouakez and Normandin (2010) or Bouakez, Chihi, and Normandin (2014) and a VAR-SV similar to those used by Cogley and Sargent (2005), Primiceri (2005) or Chan and Eisenstat (2018) to offer two common and relatively simple ways we can introduce parametric changes in volatility. However, unlike those authors, we do not pursue time-varying parameters in an effort to limit our deviation from the iid setting to changes in volatility.

We find that QVAR can statistically significantly improve over a VAR-N for forecasting density and quantiles in the tails of distributions in close to half of the variables with improvements in predictive capacity reaching between 10 and 30%. These are particularly important for labor market variables across all horizons considered and for interest and exchange rate at shorter horizons. QVAR also offers improvements over a VAR-GARCH in fewer cases concentrated in those same groups of variables. These results appear acyclic and relatively stable over time suggesting that time variation in conditional volatility and in upside and downside macroeconomic risk is generally worthwhile. Finally, QVAR almost never does statistically significantly worse than the parametric alternatives: it is therefore a robust way to model macroeconomic risk.

The second contribution of this paper is to extend the analysis to a data-rich environment by augmenting QVAR models with latent factors estimated from our set of 112 target variables. Applications featuring principal component estimates (PCA) (e.g., Manzan (2015) and Goulet Coulombe et al. (2022)) and the recently introduced iterative quantile regression (IQR) estimates of quantile factors (Chen, Dolado, and Gonzalo 2021) have been considered in the past, but all of them involve direct forecasting models in a univariate setting. In contrast, QFAVAR models the dynamic of latent factors and targets variables jointly.

We find that QFAVAR and QVAR models tend to equally well at forecasting macroeconomic risks across all variable categories. PCA and IQR factors may carry information which significantly overlaps much with the NFCI. However, QFAVAR models do provide statistically significant improvements in about 13% of cases, most of them in the labor market across all horizons. We therefore conclude from these and the previous results that QVAR and QFAVAR models are appropriate tools for modeling macroeconomic risk.

The paper is organized as follows. Section 2 introduces the QVAR model, details some of its properties and explains how to use it for forecasting. Section 3 details the forecasting experiments, the parametric models used and the performance metrics we consider. In section 4, we compare results for the QVAR and parametric alternatives while in section 5, we compare results for the QVAR and QFAVAR models. Section 6 concludes.

2. Quantile VAR Models

The QVAR model considered in this paper has been studied for scenario analysis and structural analysis by Chavleishvili and Manganelli (2021), Montes-Rojas (2021) and Ruzicka (2021). For a $K \times 1$ vector $\boldsymbol{y_t}$ of time series, the conditional quantile $\tau_k \in [0, 1]$ of the k-th variable takes the form

$$\mathbb{Q}_{\tau_k}\left(y_{k,t}|\tilde{\boldsymbol{x}}_{\boldsymbol{t}}^{(\boldsymbol{k})}\right) = \sum_{i\leq k} a_{0,k,i}(\tau_k)\, y_{i,t} + \sum_{i=1}^{K}\sum_{j=1}^{p} a_{j,k,i}(\tau_k)\, y_{i,t-j} + \epsilon_k(\tau_k) \tag{1}$$

where $\tilde{\boldsymbol{x}}_{\boldsymbol{t}}^{(\boldsymbol{k})}$ contains the regressors for this equation. It is well known in this literature that quantile regressions admit a (restricted) random coefficient representation whereby data can be simulated by uniformly sampling parameters over a grid of quantiles one equation at a time, one period at a time. This leads to

$$\boldsymbol{y_t} = \sum_{i \leq k} a_{0,k,i}(u_{k,t})\, y_{i,t} + \sum_{i=1}^{K} \sum_{j=1}^{p} a_{j,k,i}(u_{k,t})\, y_{i,t-j} + \epsilon_k(u_{k,t})$$

$$\Leftrightarrow\ \boldsymbol{y_t} = \boldsymbol{A_0}(\boldsymbol{u_t})\, \boldsymbol{y_t} + \sum_{j=1}^{p} \boldsymbol{A_j}(\boldsymbol{u_t})\, \boldsymbol{y_{t-j}} + \epsilon(\boldsymbol{u_t}) \tag{2}$$

where $\boldsymbol{u_t} \sim U[0,1]^K$, $\boldsymbol{A_0}(\boldsymbol{u_t})$ is a lower triangular matrix with a null diagonal. Contemporary terms are always included to ensure coefficients across equations do not depend on multiple $u_{k,t}$'s, but are independent to obviate the need for a notion of multivariate quantiles[2] and the triangular structure simplifies estimation.

Before turning to estimation and forecasting, we consider a few properties of QVAR processes. While it might be not be obvious, equation (1) implies the support of $\boldsymbol{y_t}$ must be bounded because, otherwise, quantile crossing would occur even in large samples[3]. The condition for ensuring the process is ergodic, as well as weakly and strongly stationary is easier to interpret by looking at the model with one variable and one lag. In this case, the condition would be $\mathbb{E}\left(a_1(u_t)^2\right) < 1$ which allows for unit and explosive roots for some subsets of conditional quantiles. Finally, it should be noted that (2) admits the following SVAR representation

$$\boldsymbol{y_t} = \bar{\boldsymbol{A}}_{\boldsymbol{0}}\, \boldsymbol{y_t} + \sum_{j=1}^{p} \bar{\boldsymbol{A}}_{\boldsymbol{j}}\, \boldsymbol{y_{t-j}} + \bar{\epsilon}_{\boldsymbol{t}} \tag{3}$$

where $\bar{\epsilon}_{\boldsymbol{t}} := \left(\boldsymbol{A_0}(\boldsymbol{u_t}) - \bar{\boldsymbol{A}}_{\boldsymbol{0}}(\boldsymbol{u_t})\right) y_t + \sum_{j=1}^{p} \left(\boldsymbol{A_j}(\boldsymbol{u_t}) - \bar{\boldsymbol{A}}_{\boldsymbol{j}}\right) \boldsymbol{y_{t-j}} + \epsilon_{\boldsymbol{t}}$ and $\bar{\boldsymbol{A}}_{\boldsymbol{j}} := \mathbb{E}\left(\boldsymbol{A_j}(\boldsymbol{u_t})\right)$ under technical conditions spelled out in Proposition 1.5 of Ruzicka (2021). This establishes that VAR and QVAR processes impose the same linear functional form for conditional expectations even as QVAR would also capture such things as conditional

[2]The interested reader can also find a technical explanation in the Theorem 1 of Chavleishvili and Manganelli (2021).

[3]Quantile regression models capture changes in conditional distributions by allowing slopes to vary with regressors across quantiles. For a single regressor, this means conditional quantiles are drawing lines which aren't parallel and must cross in the support of y_t, unless it is bounded. See discussions in Koenker and Xiao (2006) and Hallin and Werker (2006) or Ruzicka (2021) for the multivariate case.

heteroskedasticity.

An interesting consequence of this relation is that when a QVAR admits a VAR representation, results in Lütkepohl (2005) concerning linear transformations of the form $\boldsymbol{F}\,\boldsymbol{y_t}$ should apply. In particular, if a large set of variables follow a QVAR(p) process (and thus a VAR(p) process), then a subset of it will generally follow a VARMA($\tilde{p}$,$\tilde{q}$) process (with possibly some heteroskedasticity or other higher order dependence). We would therefore expect that QVAR and VAR models offer similar mean forecasts.

2.1. Estimation and Forecasting

Model parameters can be estimated by linear quantile regression (Koenker and Bassett 1978) one equation at a time for a grid of quantiles. Let $\beta^{(k)}(\tau_k)$ be all parameters for regression k, including the intercept $\epsilon_k(\tau_k)$, and $\boldsymbol{x_t^{(k)}} = \left(1, \tilde{\boldsymbol{x}}_{\boldsymbol{t}}^{(k)\prime}\right)'$ be the corresponding vector of regressors. Then the estimator is given by

$$\hat{\beta}^{(k)}(\tau_k) := \underset{\beta \in \mathbb{R}^{(k+K\,p)}}{\operatorname{argmin}} \sum_{t=p+1}^{T} \rho_{\tau_k}\left(y_{k,t} - \beta' \boldsymbol{x_t^{(k)}}\right) \tag{4}$$

where $\rho_{\tau_k}(u) := u(\tau_k - \mathbb{I}\{u < \tau_k\})$ is the quantile loss function. Under some technical conditions which guarantees among other things that the process is strongly stationary and ergodic, Ruzicka (2021) has established the asymptotic normality of this estimator[4]. This estimator further enjoys a similar property to ordinary least squares under misspecification as it offers the optimal linear approximation to conditional quantiles in a weighted least square sense (Angrist, Chernozhukov, and Fernández-Val 2006). This 'robustness' property is one of the primary motivations behind its use for macroeconomic risk modeling.

In this paper, we produce all forecasts for QVAR models by simulating future sample paths from iteratively applying the random coefficient representation (2) using estimates obtained from (4). Specifically, at each point in time the parameters are selected by choosing the point on the quantile grid that falls closest to a uniform random draw $u_{k,t}$ for each equation k. Iterating this forward allows us to draw a sample path for $\boldsymbol{y_{t+1}}, \ldots, \boldsymbol{y_{t+12}}$ and repeating this a large number of times allows us to compute a variety of statistics at a each point in time (quantile forecasts, mean forecasts, etc.).

[4] Using weights based on its asymptotic covariance matrix, $\hat{\beta}^{(k)}(\tau_k)$ viewed as a process over $\tau_k \in [0, 1]$ converges to a $K\,p + k$-dimensional standard Brownian Bridge. The interested reader can also find some results for the quantile regression estimator under unit roots and cointegration in Koenker and Xiao (2004), Xiao (2009) or Cho, Kim, and Shin (2015).

This algorithm contrasts with the approach introduced by Adrian, Boyarchenko, and Giannone (2019) in a univariate context whereby the skewed t distribution of Azzalini and Capitanio (2003) is fitted to closely match a handful of conditional quantile forecasts produced using quantile regression estimates. On the other hand, it is closer in spirit to the method used by Chavleishvili and Manganelli (2021) for stress testing and Chavleishvili et al. (2021) for risk management in a macroprudential context as we can condition forecasts on scenarios by simply imposing predetermined sequences of quantiles. It also mirrors Ruzicka (2021)'s approach for obtaining quantile impulse responses. Considering this is how the QVAR model was introduced, we limit our attention to this approach.

An important detail concerns the choice of a grid of quantiles. We opted to use a relatively fine grid of 100 equally spaced quantiles, but note that some of those quantiles may not be well estimated. Chernozhukov, Fernández-Val, and Kaji (2017) suggested using extreme value methods for quantiles beyond $\tau T/(Kp+K) \leq 15$ where $Kp+K$ is the number of parameters in the last equation. For example, a bivariate QVAR with a single lag estimated on 400 observations gives us the interval [0.15, 0.85] whereas adding a second lag reduces it to [0.225, 0.775]. Parsimony may thus be even more important when dealing with quantiles in the tails. For this reason, we follow Chavleishvili and Manganelli (2021) and Chavleishvili et al. (2021) and use a QVAR with one lag throughout the paper.

3. Forecasting Experiment

In this section, we conduct an out-of-sample forecasting experiment in which we target many monthly US variables obtained from the FRED-MD data set (McCracken and Ng 2016) spanning the period between January 1959 to June 2022. We also use the National Financial Conditions Index (NFCI) observed from January 1971 to June 2022 as a predictor in many models. Hence, to obtain a balanced panel of forecasting performance metrics, we selected all target variables from FRED-MD which started at least as early as the NFCI and did not feature any missing values in the July 2022 version of the data set. This leaves us with a subset of 112 target variable. To obtain many cycles of recessions and expansion, we set the start of the out-of-sample period to January of 1980 which gives 6 NBER recessions and a total of 510 periods for model comparison.

All target variables are transformed to induce stationarity[5] and we target the re-

[5]We follow McCracken and Ng (2016), except that we do not take second differences on interest rates,

sulting values in $h = 1, \ldots, 12$ months rather than h period averages as forecasts are produced iteratively through simulations for all models[6]. Finally, given our focus on forecasting tails, a difficult balance must be struck between allowing a large sample size for estimation and allowing the model to adapt to structural changes. We opted for a rolling window of 400 observations, allowing the window to initially expand to this size to include the recessions from the early 1980s in the analysis.

3.1. Models

The forecasting experiment includes four bivariate models with the targeted variable ordered first, followed by the NFCI. These models are the QVAR, as well as three parametric alternatives: a VAR-N, a VAR-GARCH and VAR-SV. The VAR-N is a useful benchmark insofar as it is not obvious modeling moments beyond the mean is meaningful for macroeconomic data (Plagborg-Moller et al. 2020). The VAR-GARCH and VAR-SV models are interesting as common tools in the structural VAR literature which relaxes the iid assumption of the VAR-N by allowing conditional volatility to change over time. We further consider four additional variations on the baseline QVAR model by introducing latent factor and latent quantile factor estimates as regressors, a set of hitherto unexplored extensions we call a factor augmented QVAR or QFAVAR.

VAR-N. This model takes the form

$$\boldsymbol{y_{t+1}} = \boldsymbol{\nu} + \boldsymbol{A_1}\boldsymbol{y_t} + \boldsymbol{u_{t+1}}, \quad \boldsymbol{u_{t+1}} \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}). \tag{5}$$

We estimate mean parameters $\boldsymbol{\nu}$ and $\boldsymbol{A_1}$ by ordinary least squares. The covariance matrix of innovations is estimated as $\hat{\boldsymbol{\Sigma}} = \sum_{t=2}^{T} \hat{\boldsymbol{u}}_t \hat{\boldsymbol{u}}_t' / (T - Kp - 2)\ \hat{\boldsymbol{u}}_t$ where $K = 2,\ p = 1$ and $\hat{\boldsymbol{u}}_t$ are residuals.

VAR-GARCH. We follow the structural VAR literature (e.g., Normandin and Phaneuf (2004); Bouakez and Normandin (2010); Bouakez, Chihi, and Normandin (2014)) and create a multivariate GARCH process by imposing that each 'structural' shock follows its own GARCH(1,1) process. Hence, we replace the normal for the vector of innovations

unemployment rates, monetary aggregates and prices as in Bernanke, Boivin, and Eliasz (2005). All transformations are given in Table A3 Appendix.

[6] Results in Goulet Coulombe et al. (2021) suggests averaging single period forecasts *ex post* is generally preferable to directly targeting averages when point forecasts are of primary interests, but this question lies beyond the scope of this paper.

with

(6)
$$\begin{aligned}\boldsymbol{u_{t+1}} &= \boldsymbol{D\epsilon_{t+1}}\\ \epsilon_{k,t+1} &= \sqrt{h_{k,t+1}}z_{k,t+1}, \quad z_{k,t+1} \sim N(0,1)\\ h_{k,t+1} &= (1-\alpha_k-\beta_k)+\alpha_k\epsilon^2_{k,t}+\beta_k h_{k,t}.\end{aligned}$$

where $\boldsymbol{D}$ is lower triangular to use the same restriction as in the QVAR model. We use the same parameter estimates for $\boldsymbol{\nu}$ and $\boldsymbol{A_1}$ and $\boldsymbol{\Sigma}$ as we do for the Gaussian VAR case. $\hat{\boldsymbol{D}}$ is obtained from a Cholesky factorization of $\hat{\boldsymbol{\Sigma}}$. Series of 'structural residuals' $\hat{\boldsymbol{\epsilon}}_{\boldsymbol{k,t}}$ are then obtained on which individual GARCH(1,1) processes are fitted by maximum likelihood.

(Bayesian) VAR-SV. We use one of the restricted models featured in Chan and Eisenstat (2018) which essentially replaces individual GARCH processes featured in the VAR-GARCH shown above by (random walk) stochastic volatility processes.

(7)
$$\begin{aligned}\boldsymbol{B_0}\,\boldsymbol{y}_{t+1} &= \mu+\boldsymbol{B_1}\,\boldsymbol{y}_t+\epsilon_{t+1}\\ \epsilon_{k,t+1} &= \exp\left(h_{k,t+1}/2\right)z_{k,t+1}, \; z_{k,t+1}\sim N(0,1)\\ h_{k,t+1} &= h_{k,t}+\sigma_k\zeta_{k,t+1}, \; \zeta_{k,t+1}\sim N(0,1)\end{aligned}$$

We impose recursive short-run restrictions as with the QVAR and VAR-GARCH models such that $\boldsymbol{B}_0$ is set to a lower triangular matrix with a unit diagonal. It is a common choice (e.g., Cogley and Sargent (2005) and Primiceri (2005)). The model is estimated using Bayesian methods with the following priors:

$$\theta := \left(\boldsymbol{vec}\left((\mu,\boldsymbol{B_1})'\right)', b_{0,2,1}\right)' \sim N(\boldsymbol{b}_\theta,\boldsymbol{V}_\theta), \quad \boldsymbol{h_0}\sim N(\boldsymbol{b_h},\boldsymbol{V_h}) \text{ and } \sigma_k\sim IG(\nu_k,S_k).$$

We set $\boldsymbol{b}_\theta$ and $\boldsymbol{V}_\theta$ as a Minnesota-type prior with common hyperparameter values centered on a random walk, except for the growth rates of consumption, exchange rates and stock market indexes which we center on white noise. We center the value for $b_{0,2,1}$ at 0 with a relatively large variance of 10 and likewise for the initial log variance ($\boldsymbol{b_h}=\boldsymbol{0}$ and $\boldsymbol{V}_h = 10$) following Chan and Eisenstat (2018). We use the shape $\nu_k = 5$ and scale $S_k = 0.1(\nu_k - 1)$ as Chan and Eisenstat (2018) reflecting a relatively diffuse prior centered on a small value (here, 0.1).

Their Gibb Sampling algorithm has two particular features. First of all, it jointly samples mean parameters θ for each equation whereas other algorithms would sample free elements in $\boldsymbol{B_0}$ separately. Second of all, while it applies the common auxiliary mixture sampler proposed by Kim, Shephard, and Chib (1998) which allows using methods for linear Gaussian state-space models, it samples the sequence of log variances $(\boldsymbol{h_t})_{t=1}^{T}$ in a single step for each equation using the precision sampler of Chan and Hsiao (2014). These features make the algorithm fairly efficient.

QFAVAR. As a means of exploring the value of a data-rich environment for macroeconomic forecasting, we introduce latent factor estimates as part of the vector of variables $\boldsymbol{y_t}$ in (2). This is similar in spirit to the FAVAR model of Boivin and Ng (2005), although we do not impose restrictions that would strictly justify treating the target variable and NFCI as 'observed' factors. In all cases, latent factors are recursively estimated using the in-sample data window. We collect our 112 variables into a matrix X and let variable i obey

$$x_{i,t} = \boldsymbol{\lambda_i'} \boldsymbol{f_t} + \nu_{i,t} \tag{8}$$

where $\boldsymbol{f_t}$ is a $r \times 1$ vector of factors and $\boldsymbol{\lambda_i}$ is the corresponding vector of loadings. Following common practice since Stock and Watson (2002a,b), we obtain factor estimates $\boldsymbol{\hat{f}_t}$ by principal component. We set $r = 1$ factor out of concern for parsimony so our vector of time series becomes $\boldsymbol{y_t} = (y_{1,t}, \hat{f}_t, NFCI_t)'$. A natural alternative would be to consider doing the same thing using the quantile latent factors recently introduced by Chen, Dolado, and Gonzalo (2021). In this case, we have

$$\mathbb{Q}\left(x_{i,t} | \boldsymbol{f_t}(\tau)\right) = \boldsymbol{\lambda_i}(\tau)' \boldsymbol{f_t}(\tau) + \nu_{i,t}(\tau) \tag{9}$$

with $f_t(\tau)$ being $r(\tau) \times 1$. We obtain estimates $\boldsymbol{\tilde{f}_t}(\tau)$ for the 5th and 95th quantiles using the IQR algorithm (Chen, Dolado, and Gonzalo 2021). Again, we set $r(\tau) = 1$ for parsimony and use $\boldsymbol{y_t} = (y_{1,t}, \tilde{f}_t(0.05), \tilde{f}_t(0.95), NFCI_t)'$.

3.2. Forecasting Evaluation

We are primarily interested in the relative ability of each model to produce forecasts for the tails of the distribution, but also consider evaluating point forecasts. For model

m and variable v, define the h period ahead mean and quantile forecasts as

$$\hat{y}_{t+h,v,m} := \hat{\mathbb{E}}_m \left(y_{t+h,v} | \mathcal{F}_t \right) \qquad \hat{q}_{t+h,v,m}(\tau) := \hat{\mathbb{Q}}_{\tau,m} \left(y_{t+h,v} | \mathcal{F}_t \right).$$

We evaluate mean and quantile forecasts using square and quantile scores, respectively

$$L^2 \left(\hat{y}_{t+h,v,m}, y_{t+h,v} \right) := \left(y_{t+h,v,m} - y_{t+h,v} \right)^2 \tag{10}$$

$$QS_\tau \left(\hat{q}_{t+h,v,m}(\tau), y_{t+h,v} \right) := \rho_\tau \left(\hat{q}_{t+h,v,m}(\tau) - y_{t+h,v} \right). \tag{11}$$

In both cases, their use is motivated by the well-known fact that the expected values of those loss functions are minimized if the 'true' conditional expectation and quantiles are used. We follow Carriero, Clark, and Marcellino (2020) and Carriero, Clark, and Marcellino (2022) in our evaluation of density forecasts and adopt the quantile weighted continuous ranked probability score (CRPS) introduced by Gneiting and Ranjan (2011). For a grid of N quantiles, this score is defined as

$$qwCRPS \left(\hat{\boldsymbol{q}}_{\boldsymbol{t+h,v,m}}, \boldsymbol{\nu}, y_{t+h,v} \right) = \frac{2}{N-1} \sum_{j=1}^{N} \nu(\tau_j) QS_{\tau_j} \left(\hat{q}_{t+h,v,m}(\tau_j), y_{t+h,v} \right) \tag{12}$$

where $\boldsymbol{\nu} := \left(\nu(\tau_j) \right)_{j=1}^{N}$ is a vector of weights and $\hat{\boldsymbol{q}}_{\boldsymbol{t+h,v,m}} := \left(\hat{q}_{t+h,v,m}(\tau_j) \right)_{j=1}^{N}$ stacks quantile forecasts into a vector. Gneiting and Ranjan (2011) proposed using $\nu(\tau_j) = \tau_j^2$, $\nu(\tau_j) = (1 - \tau_j)^2$ and $\nu(\tau_j) = (2\tau_j - 1)^2$ to put more weight on the right tail, left tail or both tails jointly. This scoring rule is proper meaning that it would be minimized in expectation by the true conditional density (Gneiting and Raftery 2007). Note that all performance metrics are negatively oriented so that we seek to minimize them.

4. Discussion of QVAR Results

To build intuition about how the QVAR model performs relative to the parametric alternatives, we begin our discussion by focusing on a handful of target variables: the growth rate of output as measured by industrial production, the unemployment rate, the CPI inflation rate and the federal funds rate. The interest of the first three variables lies in them being common targets in this literature[7]. The federal funds rate happens

[7] E.g., Giglio, Kelly, and Pruitt (2016), Adrian, Boyarchenko, and Giannone (2021) and Plagborg-Moller et al. (2020) all looked at risk to different measures of output growth, while Kiley (2022) studied risk to

to be an example of a variable for which QVAR forecasts risk relatively well and is of independent interest as a key indicator of the stance of monetary policy.

Table 1 displays the performance of QVAR, VAR-GARCH and VAR-SV relative to the VAR-N benchmark for each of those series at aa horizon of one month over the entire out-of-sample period. Specifically, let $l_{t,v}^{(m,h)}$ be the loss of model m at horizon h, time t and variable v, the table shows

$$\bar{R}_{T,v}^{(m,h)} = \frac{\sum_{t=1}^{T} l_{t,v}^{(m,h)}/T}{\sum_{t=1}^{T} l_{t,v}^{(VAR-N,h)}/T} \tag{13}$$

where T is the size of the out-of-sample period. Significance for the difference in loss between models using a Diebold and Mariano (1995) test with HAC standard errors are also indicated. Focusing on the first column, we can see that all models show improvements over the VAR-N benchmark for output growth, inflation and the federal funds rate. QVAR and VAR-GARCH show marginally significant gains of 6 and 9% for forecasting the tail density of output growth, respectively. Tables A1 and A2 in the Appendix show the small edge QVAR enjoys for output growth persists and becomes significant at 1% at horizons of 3 and 6 months. But the largest improvements in Table 1 occur for the federal funds rate with all three models showing gains over 30%, all statistically significant at 1%.

the unemployment rate and López-Salido and Loria (2020) focused on inflation risk.

Table 1. Selected Series (All Periods at 1 month ahead)

	CRPS (Tails)	CRPS (Left)	CRPS (Right)	QS05	QS95	MSE
QVAR						
Output growth	0.942*	0.994	0.926**	1.007	0.844*	0.904
Unemployment rate	1.011	1.046	0.975	1.13	0.963	1.016
Inflation rate	0.98	0.999	0.972*	1.007	0.958	1.007
Fed. funds rate	0.633***	0.734***	0.658***	0.664***	0.591***	0.991
VAR-GARCH						
Output growth	0.916*	1.002	0.905	0.954	0.725	1.003
Unemployment rate	1.016	1.005	1.019	0.949	1.026	0.992
Inflation rate	0.958***	0.984	0.972***	0.914	0.917	1.005
Fed. funds rate	0.635***	0.739***	0.691***	0.625***	0.521***	1.001
VAR-SV						
Output growth	0.931	1.011	0.896*	0.986	0.738	0.889
Unemployment rate	1.003	0.989	1.014	0.972	1.03	1.018
Inflation rate	0.957***	0.994	0.953***	0.952	0.891*	1.008
Fed. funds rate	0.603***	0.717***	0.637***	0.632***	0.514***	0.987

The ratios of mean predictive metric indicate the model beats the VAR-N benchmark when below one. Statistical significance at the 10% (*), 5% (**) and 1% (***) levels for the Diebold and Mariano (1995) test using Newey and West (1987) standard errors are also shown. QS refers to the quantile score and CRPS to the indicated quantile weighted CRPS.

Tables A1 and A2 in the Appendix show these improvements diminish at the 3 months market to a little over 20% while remaining statistically significant, but almost disappear at the 6 months mark with only the QVAR showing marginally statistically significant improvements a little over 10%. Single tail weighted CRPS and quantile score metrics show equally large and statistically significant gains in both tails for the federal funds rate.

A natural question to ask is whether this a cyclical aspect to these gains. We may believe most of the gains are concentrated during recessions or at least around them. Figures 1 and 2 compares the QVAR and VAR-N forecasts for the mean, the 5th and the 95th quantile for the federal funds rate and growth rate of output at the 1 month horizon for periods of 24 months surrounding each of the 6 NBER recessions we cover.

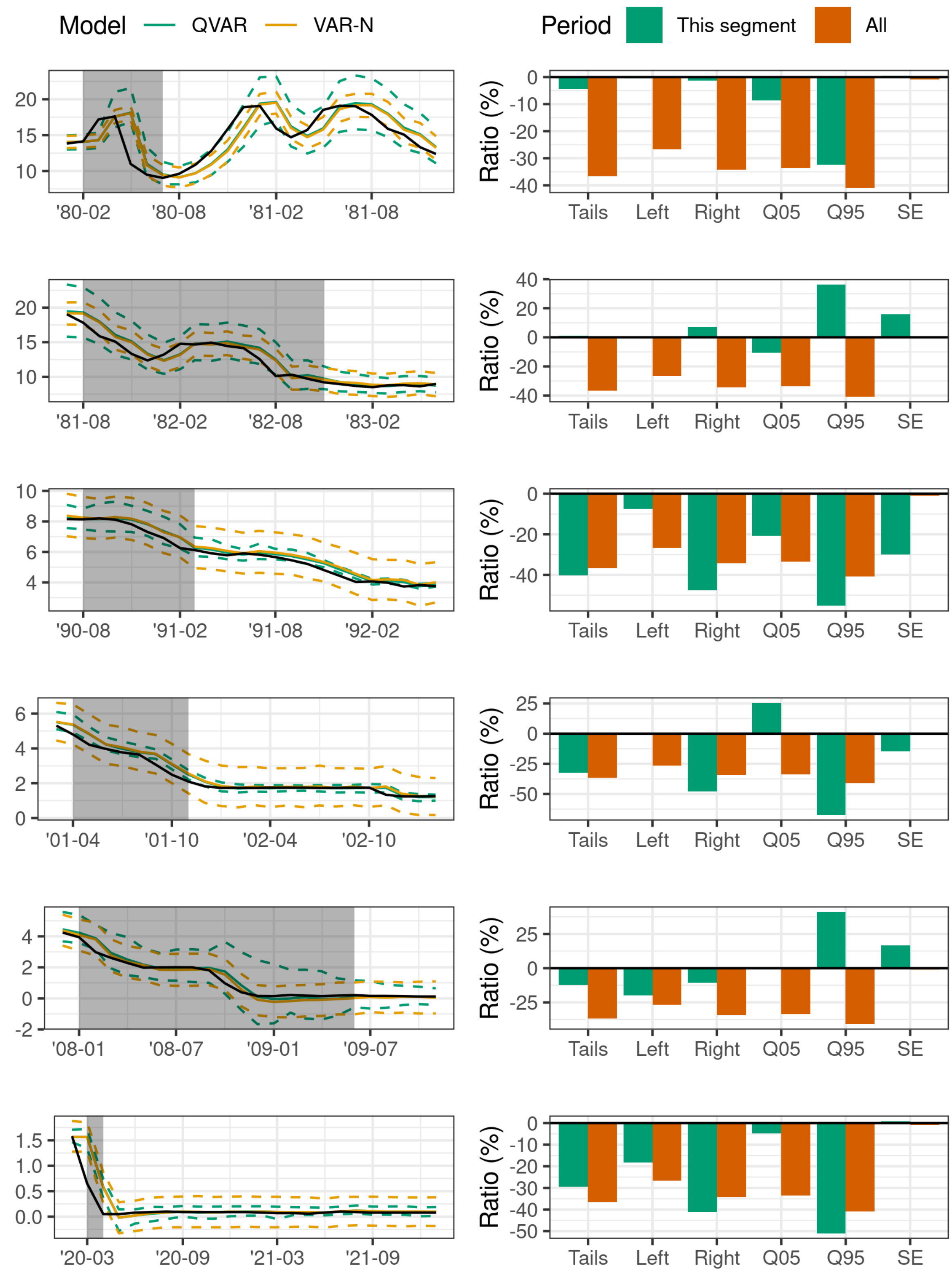

FIGURE 1. Recessions (Federal funds rate, 1 month ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

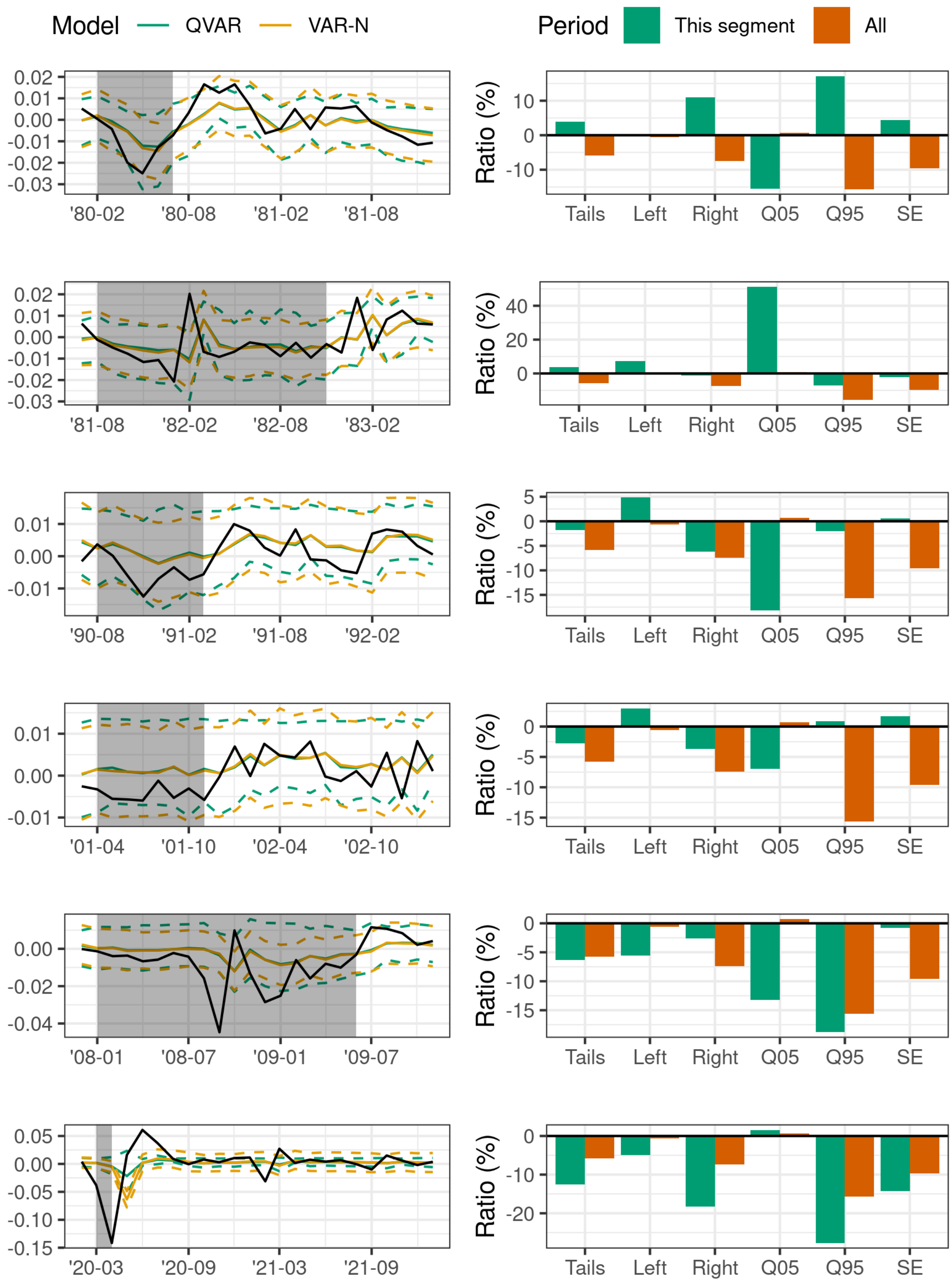

FIGURE 2. Recessions (Output growth, 1 month ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

Each figure also reports performance ratios as in equation (13) for all of the out-of-sample period, as well as for each of the 24 months segments. They have been centered on zero so values below zero indicate the QVAR outperforms the VAR-N. There's a general pattern of QVAR doing better than VAR-N across all metrics with minor exceptions for the federal funds rate. It also appears most of the advantage isn't located during recessions, but after recessions where QVAR provides much tighter intervals. Given that VAR-GARCH and VAR-SV perform similarly per Table 1, it is possible that allowing for variation in the second moment is what matters here. Results for output growth also shows QVAR does not perform better than VAR-N during recessions, but sometimes get an advantage during the recovery. This has been the case in most metrics since the 2001 recession. Figures A1 through A6 in the Appendix show results for the other two variables and a horizon of 6 months. It's visible that QVAR's advantage over VAR-N is mostly about doing better at capturing time varying risks outside of recession periods.

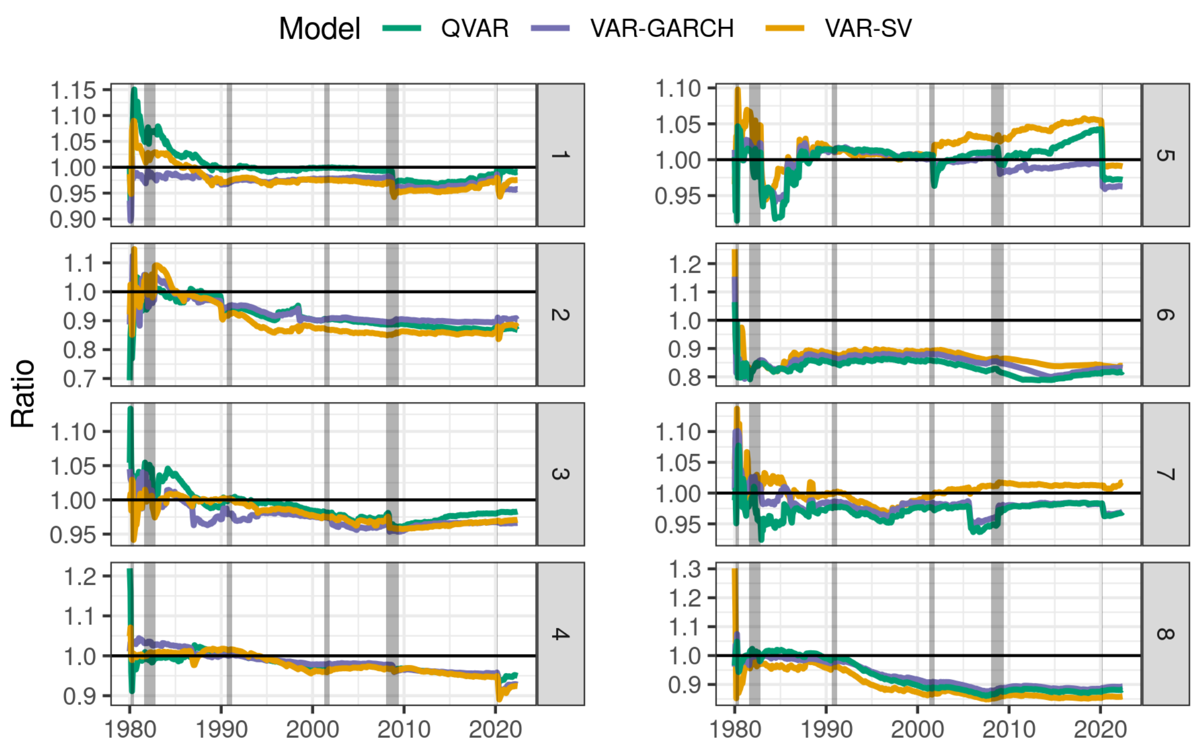

FIGURE 3. Recursive average ratios to VAR-N (Tails weighted CRPS, 1 month ahead)

The figure features the median of averages across variables in each group. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

To see how well this impression that QVAR may be beneficial for modeling risks outside of recession generalize, we now consider computing performance ratios recursively (that is, $\bar{R}^{(m,h)}_{T^*,v}$ for $T^* = 1, \ldots, T$) for each of our 112 variables. Figure 3 displays the time series of median values of $\bar{R}^{(m,h)}_{T^*,v}$ for each of the 8 categories of variables included in FRED-MD[8]. Values below unity indicate a given model outperformed VAR-N. We see that all three models perform remarkably similarly and do especially well for the labor market (group 2), the interest and exchange rates (group 6) and the stock market (group 8) at a horizon of 1 month. And, again, relative performance isn't consistently improved during recessions, even for income and output (group 1), although it is true for the 2008 recession. This is unsurprising since this followed from financial crisis and we use the NFCI as in Adrian, Boyarchenko, and Giannone (2019). Figures A7 and A8 in the Appendix show the results for horizons of 3 and 6 months. They reinforce the same point that QVAR doesn't get most of its edge from recessions. It also illustrates that model performance is relatively stable over time[9].

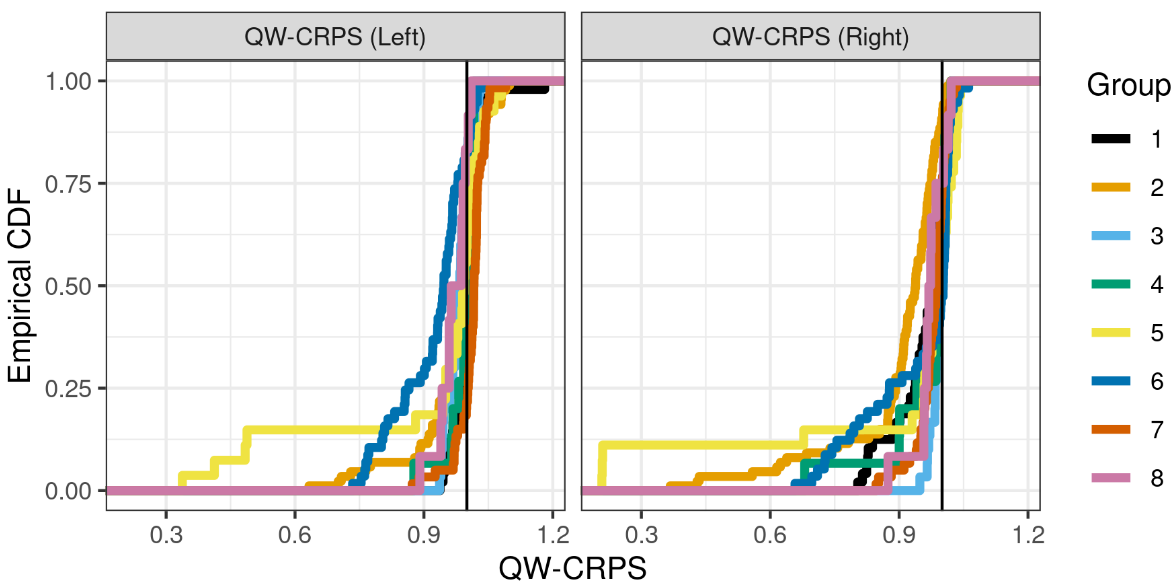

FIGURE 4. 1 Month ahead quantile weighted CRPS ratios (QVAR to VAR-N)

Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

[8]There are 112 series of recursive ratios, $\bar{R}^{(m,h)}_{T^*,v}$, for each pair of models and horizon. We opted to visualize the median to avoid the handful of cases where VAR-SV performs poorly such as on oil prices.

[9]VAR-GARCH and VAR-SV do provide an exception at 3 and 6 months where they clearly gained most of their grounds during the COVID recession for labor market variables.

Now that we have a sense of how risk forecasting performances evolve over time, we return to comparisons involving all of the out-of-sample period. Figure 4 displays group-wise distributions of ratios of mean right and left weighted CRPS for the QVAR relative to the VAR-N benchmark[10]. As before, values below unity mean that the QVAR model outperforms the VAR-N benchmark. The previous figure focused on how on the median of each of those distributions evolved. Here we're interested on seeing the magnitude and frequencies of improvements.

We can see that on the right and left tails at the one month horizon, QVAR provides improvement in excess of 10% for 25% of interest and exchange rate variables (group 6), 15% of money and credit variables (group 5) and around 10% of the labor market (group 2) variables. Some of the gains for money and credit, as well as labor market variables are very substantial, exceeding 30% improvements. Figures A9 and A10 in the Appendix show gains for increasingly fewer cases at the 3 and 6 months horizons, respectively. That being said, the handful of large improvements for labor market and money and credit variables appear to subsist at those medium term horizons. Figures A11 through A13 in the Appendix show the picture is robust to using quantile scores rather than quantile weighted CRPS as performance metrics. In other words, gains to modeling changes in tail risk persist for longer when looking at those macroeconomic variables rather than financial variables like interest and exchange rates where the gains are shorter-lived.

While we see QVAR can substantially outperform VAR-N and does so in many cases, we have yet to determine whether the differences in performance are statistically significant. We proceed by using a Diebold and Mariano (1995) test to categorize variables in each of the 8 groups featured in FRED-MD. We say that QVAR wins over a given benchmark model if it has a lower average loss and the difference is significant at 5%. Likewise, we say that QVAR loses if it has a higher average loss and the difference is significant at 5%. In other cases, models are classified as equally good. We categorize all variables in this manner for forecasts at horizons of 1 to 12 months.

Figure 5 presents these group-wise count for the tails weighted CRPS metric. The upper left panel shows that QVAR has a statistically significantly improves over the VAR-N benchmark for nearly 60 out of 112 variables at shorter horizons meaning it's often worth going beyond iid shocks for tail density forecasting. QVAR beats the VAR-N for interest and exchange rates (group 6), as well as stock market (group 8) variables

[10]Specifically, it is the empirical CDF of $\bar{R}^{(QVAR,1)}_{T,\nu}$ from equation (13) for each group of variables in FRED-MD.

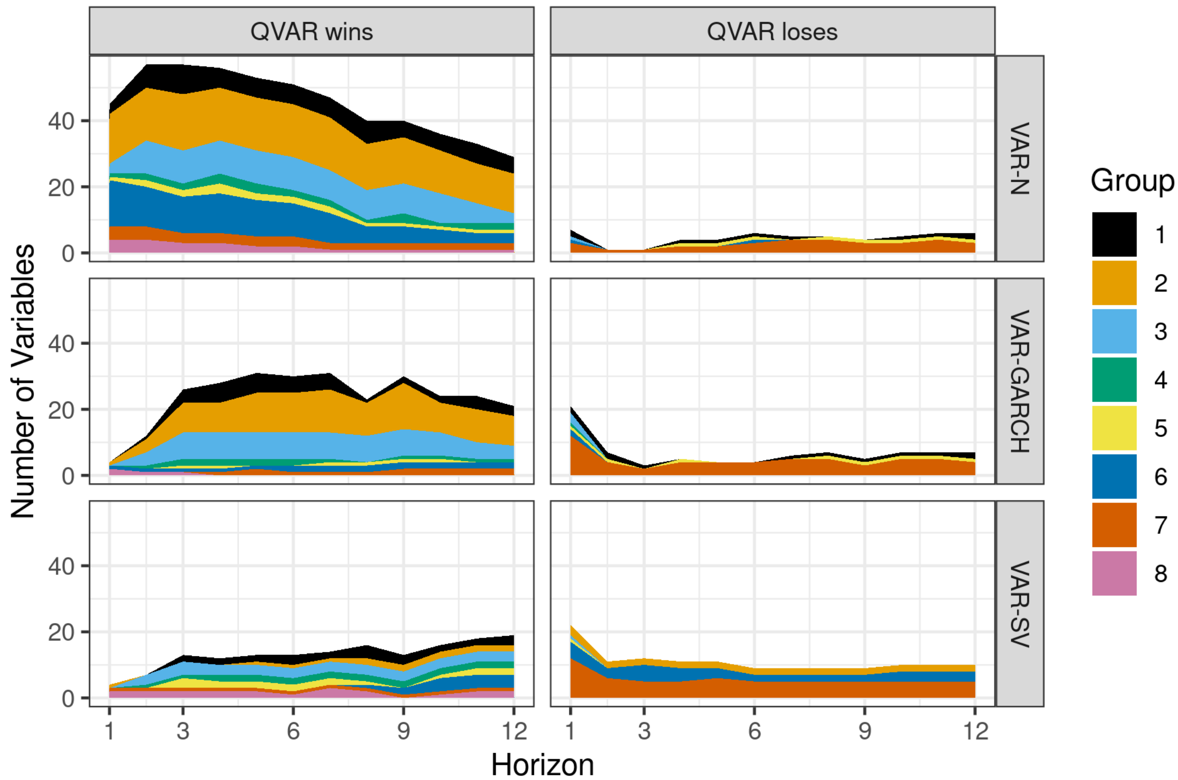

FIGURE 5. QVAR and benchmark model comparison (Tails weighted CRPS)

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

at short horizons. QVAR performs better than both VAR-N and VAR-GARCH for about 1/3 of labor market (group 2) variables starting at about 3 months ahead going forward. QVAR rarely loses to parametric alternatives, except for prices (group 7).

QVAR beats the other two parametric alternatives in fewer cases. Part of that fall comes from interest and exchange rates (group 6), suggesting QVAR improves over VAR-N in this category by enabling for changes in volatility like VAR-GARCH and VAR-SV. Another part of the fall vis-à-vis VAR-SV, but not VAR-GARCH is the labor market (group 2). The variables related to unemployment and hours worked in that group are very persistent, The Minnesota prior used by the VAR-SV therefore suggest QVAR improves over the OLS estimates used by VAR-N and VAR-GARCH on persistent data. This is visible in Figure A14 of the Appendix which displays the comparisons for square loss.

In summary, QVAR improves upon VAR-N in about half of all variables and rarely does more poorly than any of the parametric alternatives. QVAR appears to be especially well suited to model the labor market, as well as interest and exchange rates. The gains do not appear tied to recessions, seem stable over time and reaching between 10 and 30% in many cases. Moreover, large gains are statistically significant. Finally, QVAR almost never does statistically significantly worse than the parametric alternatives: it is therefore a robust way to model macroeconomic risk.

5. Discussion of QFAVAR Results

In this section, we consider the usefulness of extending the QVAR model to a data-rich environment by alternatively adding PCA and IQR latent factor estimates to our bivariate QVAR.

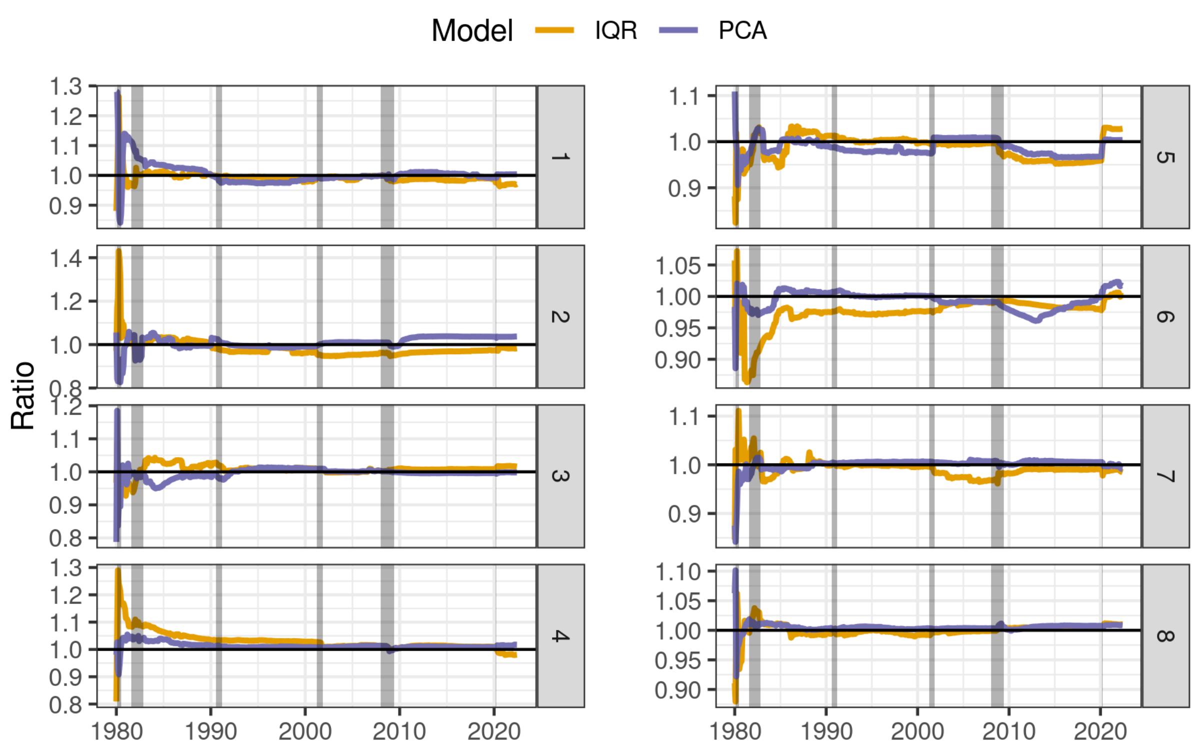

FIGURE 6. Recursive average ratios to QVAR (Tails weighted CRPS, 1 month ahead)

The figure features the median of averages across variables in each group. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

Figure 6 illustrates the evolution over time of the predictive performance of QFAVAR relative to QVAR for tail densities. As in Figure 3, we recursively compute ratios of average tail weighted CRPS according to equation (13) for all variables and display the group-wise median. Values below unity mean that the QFAVAR model outperforms the QVAR model. QFAVAR models perform similarly to QVAR models for tail density forecasting. This feature is stable over time across all groups. Figure A17 in the Appendix illustrates the same point at the 6 months horizon.

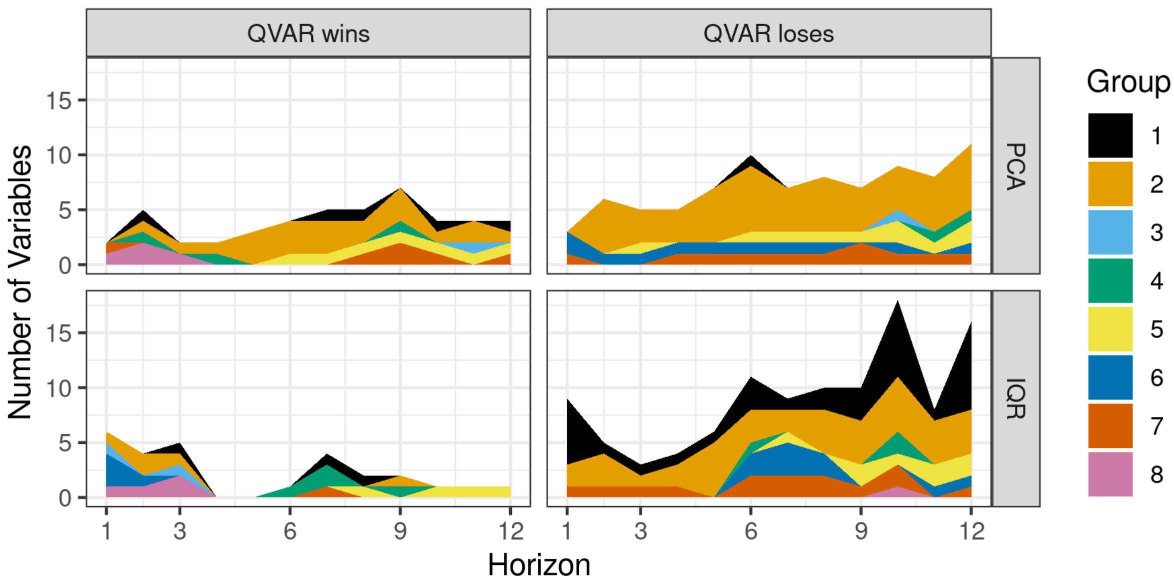

FIGURE 7. QFAVAR and QVAR comparison (Tails weighted CRPS)

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

As one might suspect from those observations, once we turn to formal tests, the differences in scores between QVAR and QFAVAR models aren't statistically significant in most cases. Figure 7 displays those results for formal tests over the entire out-of-sample period for tail weight CRPS. As in the previous section, we say that QVAR wins for a given variable if its average score is lower than for QFAVAR and the difference is statistically significant at 5% using a Diebold and Mariano (1995) test. A similar definition applies to cases where QVAR loses (QFAVAR wins) and, otherwise, we deem that both models perform equally well.

One of the patterns which holds across all QFAVAR models is that adding factors pays off more often at longer horizons. Another one is that QVAR and QFAVAR models perform equally well at density forecasting for about 100 of the 112 target variables in our sample. Given that the NFCI is estimated using a combination of macroeconomic and financial variables, it is possible that it shares some information with the PCA and IQR factors we extracted. Adding PCA or IQR factors improves tail density forecasts for some of the labor market (group 2) variables even though QVAR already performed well in this category relative to parametric alternatives. IQR helps with output and income (group 1) in a handful of cases in a way that PCA does not. Figures A20 and A18 in the Appendix display the same information for square loss and right and left weighted CRPS metrics, respectively. They show the improvements made from using PCA and IQR factors in the labor market is related to improvements in forecasting conditional means. However, for income and output, gains from including IQR factors show up in right and left weighted CRPS metrics, but not in the square loss metric. This isn't particularly surprising given that we extracted IQR factors in the tails.

6. Conclusion

In this paper, we evaluated the performance of the QVAR model for forecasting macroeconomic risk. To this end, we performed a large out-of-sample forecasting experiment on US monthly variables using VAR-N, VAR-GARCH and VAR-SV models as parametric benchmarks. All models were specified as bivariate models featuring the target variables and the NFCI.

We found that QVAR statistically significantly outperforms VAR-N in about half of our variables across horizons of 1 to 12 months for forecasting densities or quantiles in the tails. It works particularly well for the labor market at all horizons, as well as interest and exchange rates at shorter horizons. These gains are large, ranging between 10 and 30% over the VAR-N benchmark. Some of these gains appear to be tied to the ability of QVAR to approximate changes in conditional volatility. Improvements appear stable over time and are not systematically concentrated in recession periods. Finally, QVAR almost never performs significantly worse than any of the parametric alternatives. In this sense, it offers a robust approximation to conditional distributions.

We then extended QVAR to a data-rich environment by introducing PCA or IQR factors as additional predictors. The resulting QFAVAR model statistically significantly improves upon the QVAR model for forecasting macroeconomic risks in around 13% of our target variables. Most of the improvements are tied to labor market variables, especially at long horizons.

In summary, we find that QVAR and QFAVAR models are adequate tools for modeling macroeconomic risk. This is relevant from a macroprudential risk management perspective as in Chavleishvili et al. (2021). Extending the present analysis to a simulation experiment featuring DSGE models with financial frictions in particular would provide useful complementary evidence.

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Appendix A. Selected Series Results

TABLE A1. Selected Series (All Periods at 3 months ahead)

	CRPS (Tails)	CRPS (Left)	CRPS (Right)	QS05	QS95	MSE
QVAR						
Output growth	0.968***	0.998	0.954***	0.99	0.938*	0.972
Unemployment rate	1.013	1.046	0.983	1.159	0.972	1.039
Inflation rate	0.967*	0.987	0.951***	1.009	0.948	0.996
Fed. funds rate	0.753***	0.84***	0.767***	0.729**	0.687***	1.009
VAR-GARCH						
Output growth	0.99	1.013	0.977	1.017	0.929	1.005
Unemployment rate	0.975	0.97	0.996	0.854	1.003	0.982
Inflation rate	0.993	1.021*	0.973***	1.026	0.963	1.002
Fed. funds rate	0.774***	0.916	0.776***	0.821	0.561***	1
VAR-SV						
Output growth	0.988	1.032*	0.945***	1.029	0.901	0.96
Unemployment rate	0.997	0.983	1.018	0.938***	1.024	1.048
Inflation rate	0.967	1	0.936***	1.011	0.916	0.974**
Fed. funds rate	0.737***	0.867**	0.723***	0.83	0.534***	0.98

The ratios of mean predictive metric indicate the model beats the VAR-N benchmark when below one. Statistical significance at the 10% (*), 5% (**) and 1% (***) levels for the Diebold and Mariano (1995) test using Newey and West (1987) standard errors are also shown. QS refers to the quantile score and CRPS to the indicated quantile weighted CRPS.

Table A2. Selected Series (All Periods at 6 months ahead)

	CRPS (Tails)	CRPS (Left)	CRPS (Right)	QS05	QS95	MSE
QVAR						
Output growth	0.968***	1.005	0.954***	0.985	0.937**	0.994*
Unemployment rate	1.032	1.057	1	1.214	1.004	1.064
Inflation rate	0.972	0.985	0.962**	0.995	0.976	1.003
Fed. funds rate	0.851**	0.901*	0.83**	0.91	0.839	1.013
VAR-GARCH						
Output growth	0.996	1.007	0.991***	0.991	0.999	1.007*
Unemployment rate	0.981	0.977	1	0.867	1.005	0.978
Inflation rate	0.996	1.01	0.984*	1.031	0.996	1.003
Fed. funds rate	0.889	1.048	0.82***	1.121	0.581**	1
VAR-SV						
Output growth	1.011	1.043***	0.961***	1.06*	0.971	0.988**
Unemployment rate	1.02	0.997	1.042	0.984	1.057	1.087
Inflation rate	0.977	0.996	0.948**	1.009	0.954	0.976
Fed. funds rate	0.864	0.965	0.801	1.028	0.694	0.996

The ratios of mean predictive metric indicate the model beats the VAR-N benchmark when below one. Statistical significance at the 10% (*), 5% (**) and 1% (***) levels for the Diebold and Mariano (1995) test using Newey and West (1987) standard errors are also shown. QS refers to the quantile score and CRPS to the indicated quantile weighted CRPS.

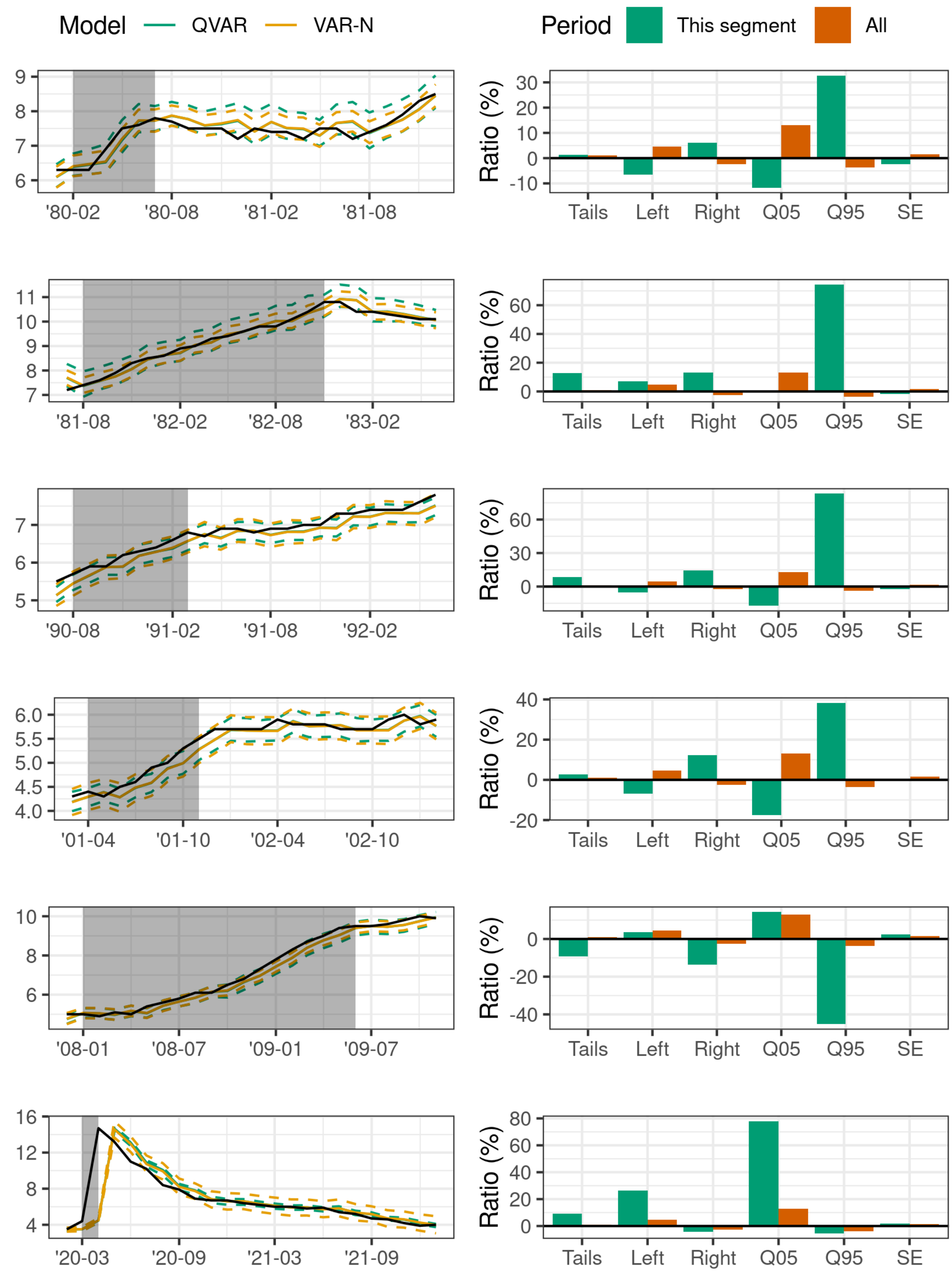


FIGURE A1. Recessions (Unemployment rate, 1 month ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

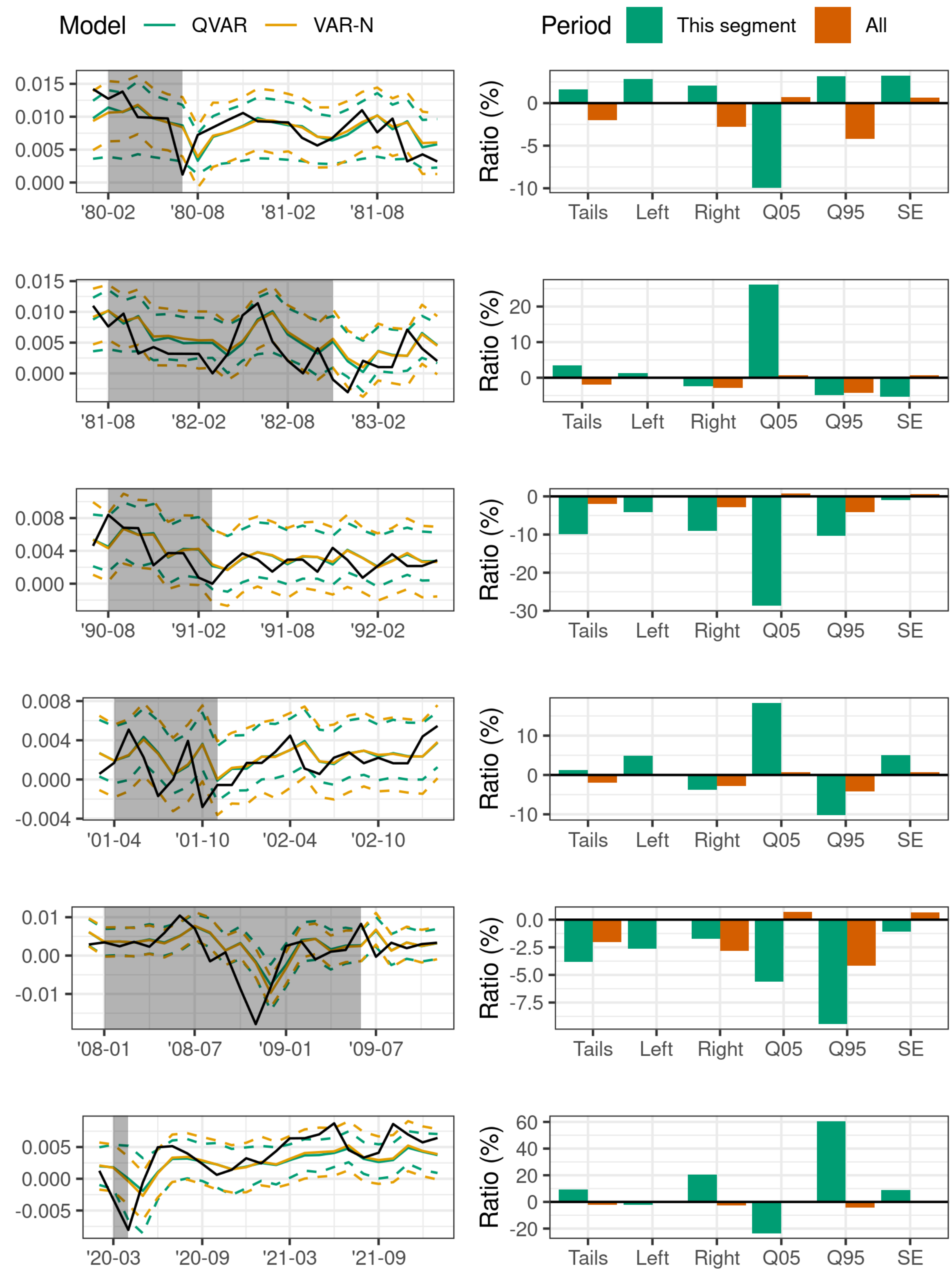


FIGURE A2. Recessions (Inflation rate, 1 month ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

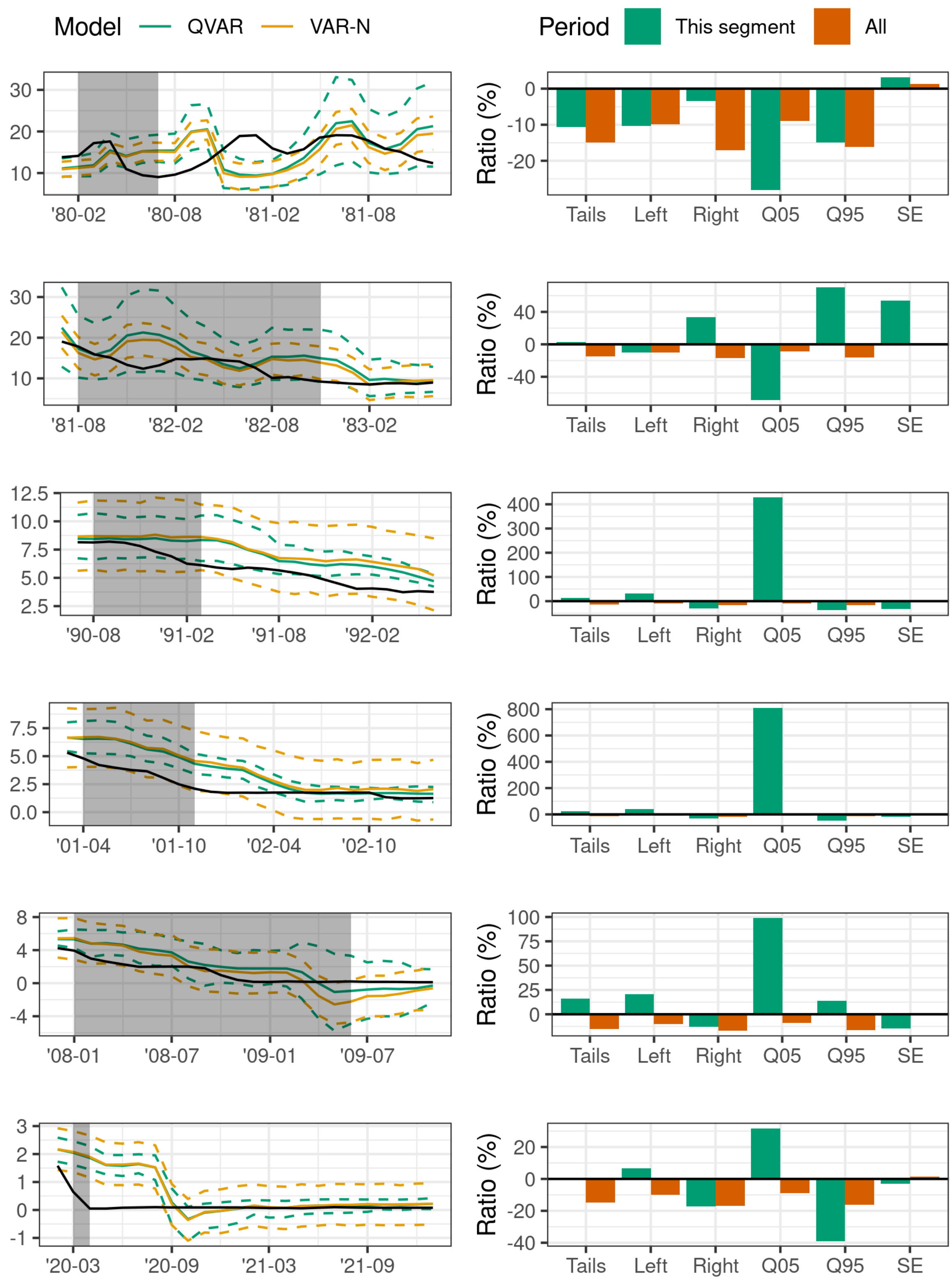


FIGURE A3. Recessions (Federal funds rate, 6 month ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

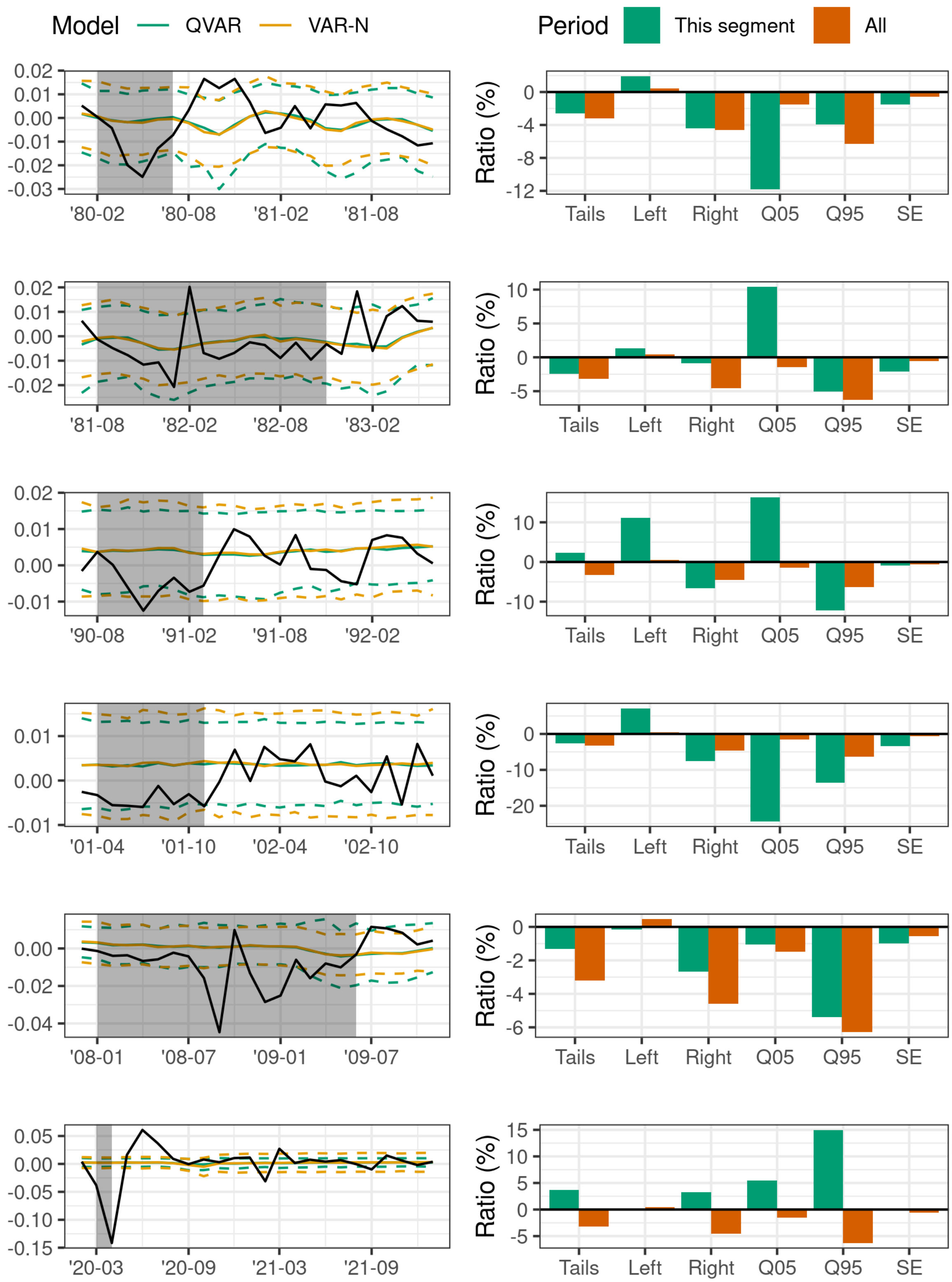


FIGURE A4. Recessions (Output growth, 6 month ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

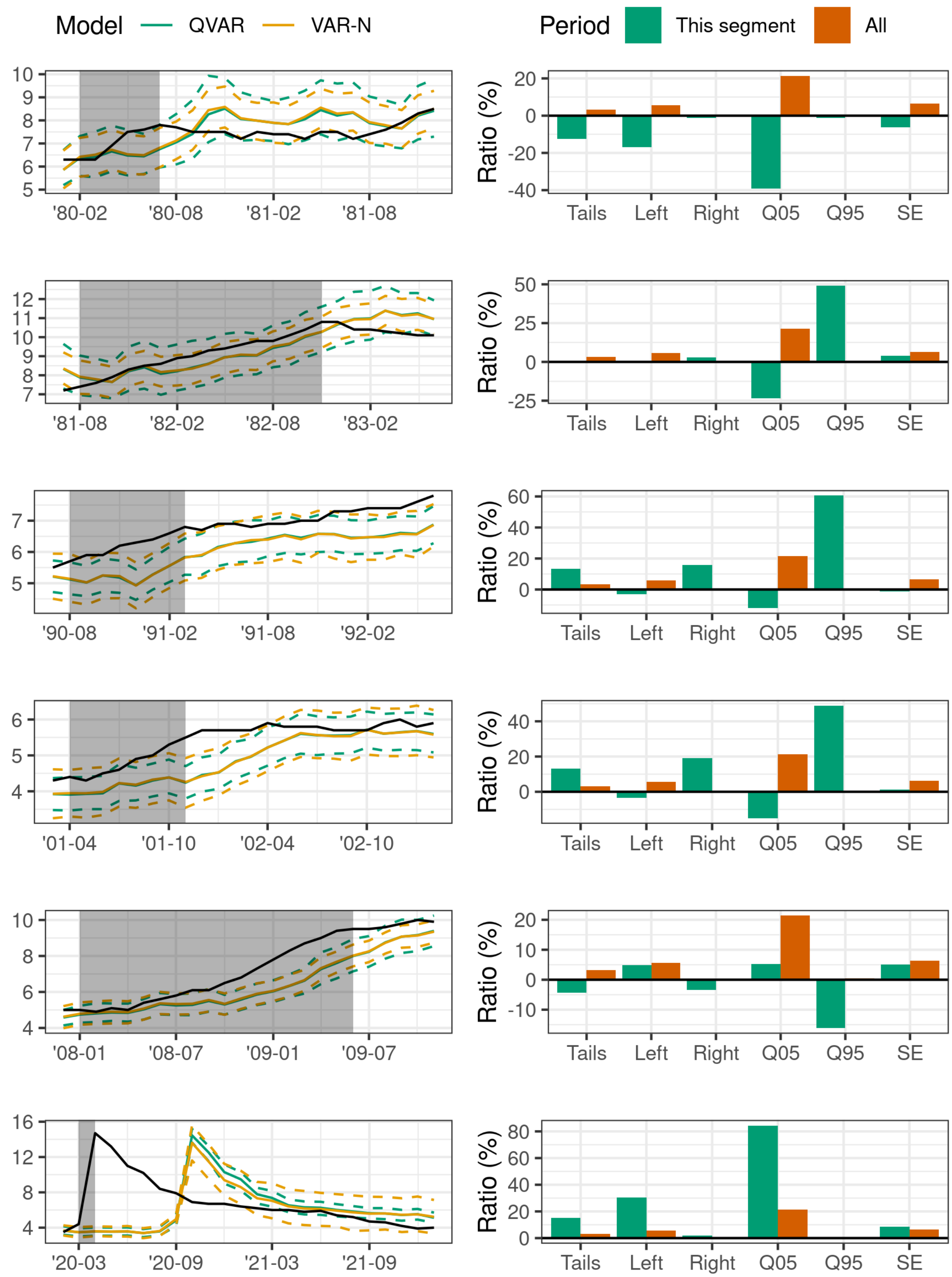


FIGURE A5. Recessions (Unemployment rate, 6 months ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

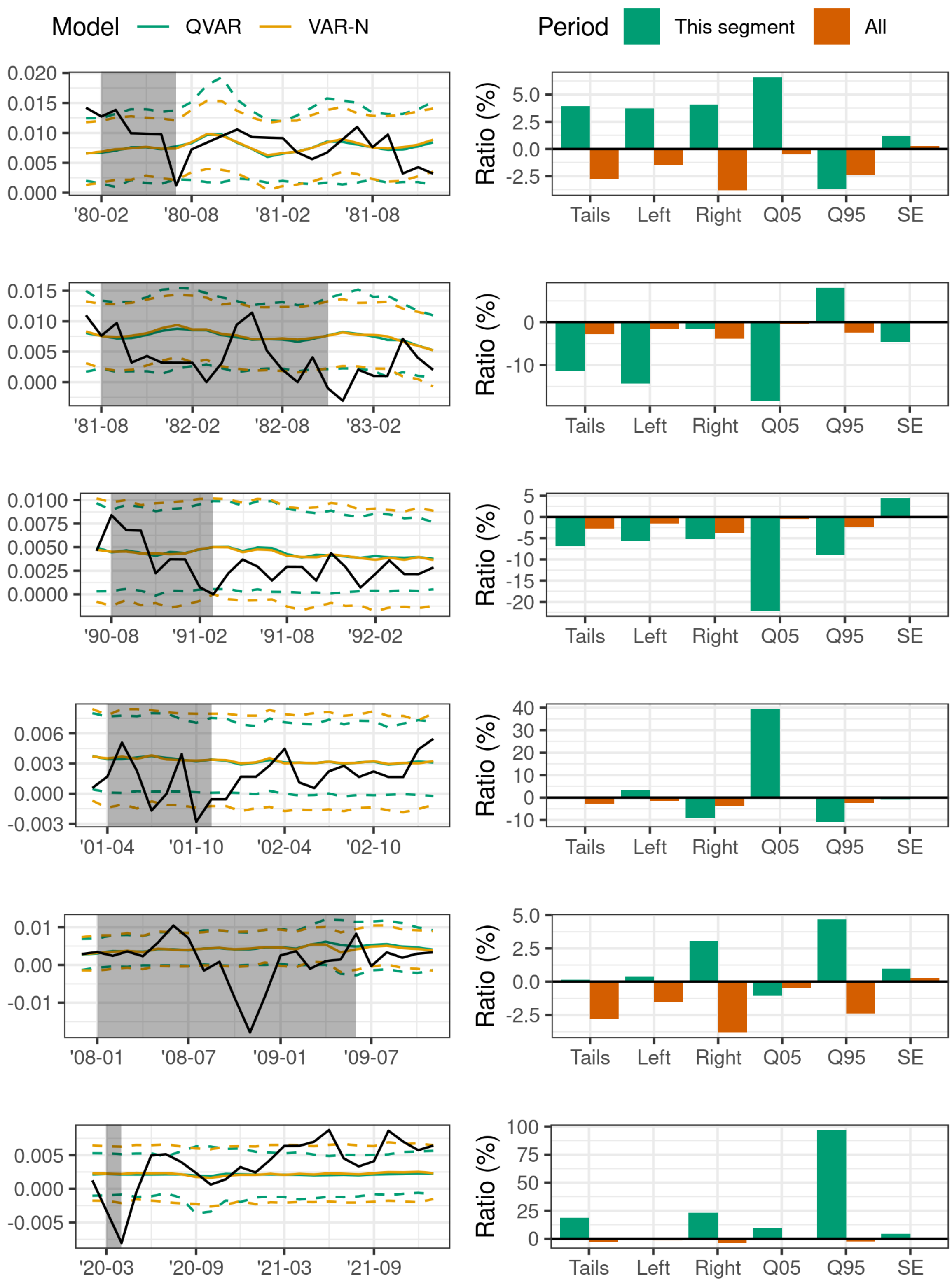


FIGURE A6. Recessions (Inflation rate, 6 months ahead)

Ratios show the percentage different between QVAR and VAR-N with negative indicating improvements. NBER recessions are shaded. SE: squared error.

Appendix B. Group-Wise Results

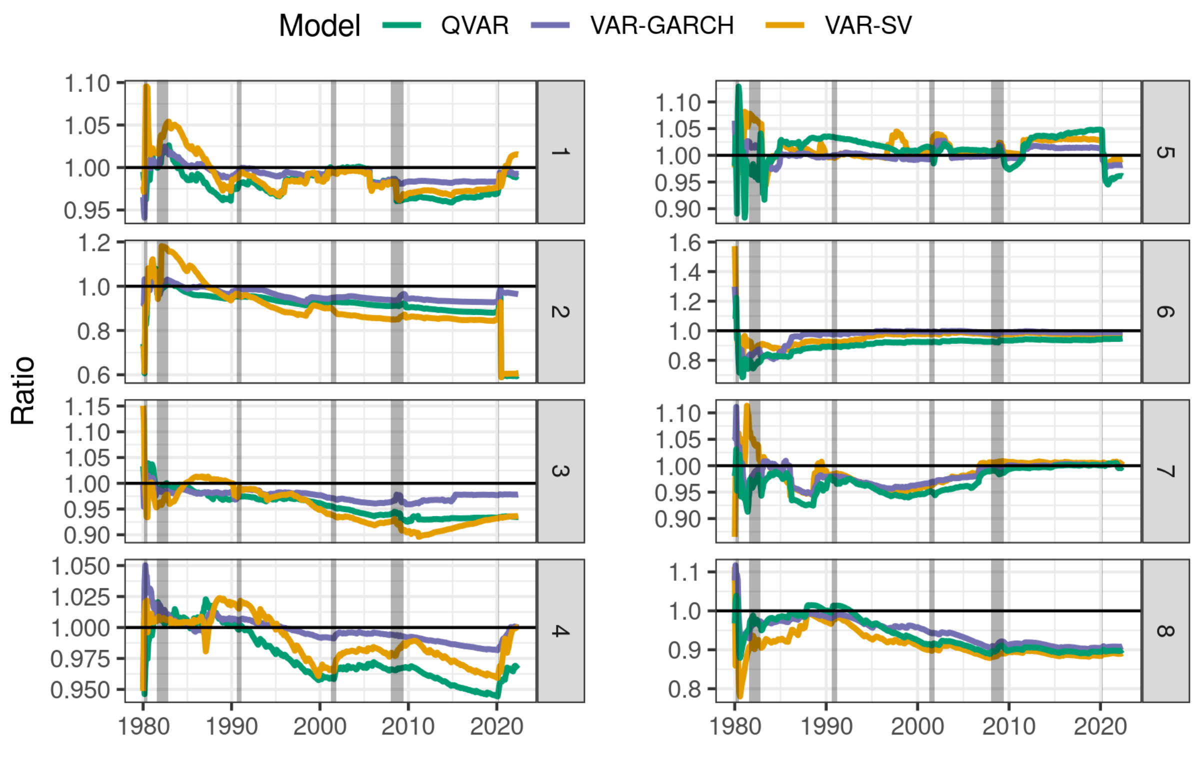


FIGURE A7. Recursive average ratios to VAR-N (Tails weighted CRPS, 3 months ahead)

The figure features the median of averages across variables in each group. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

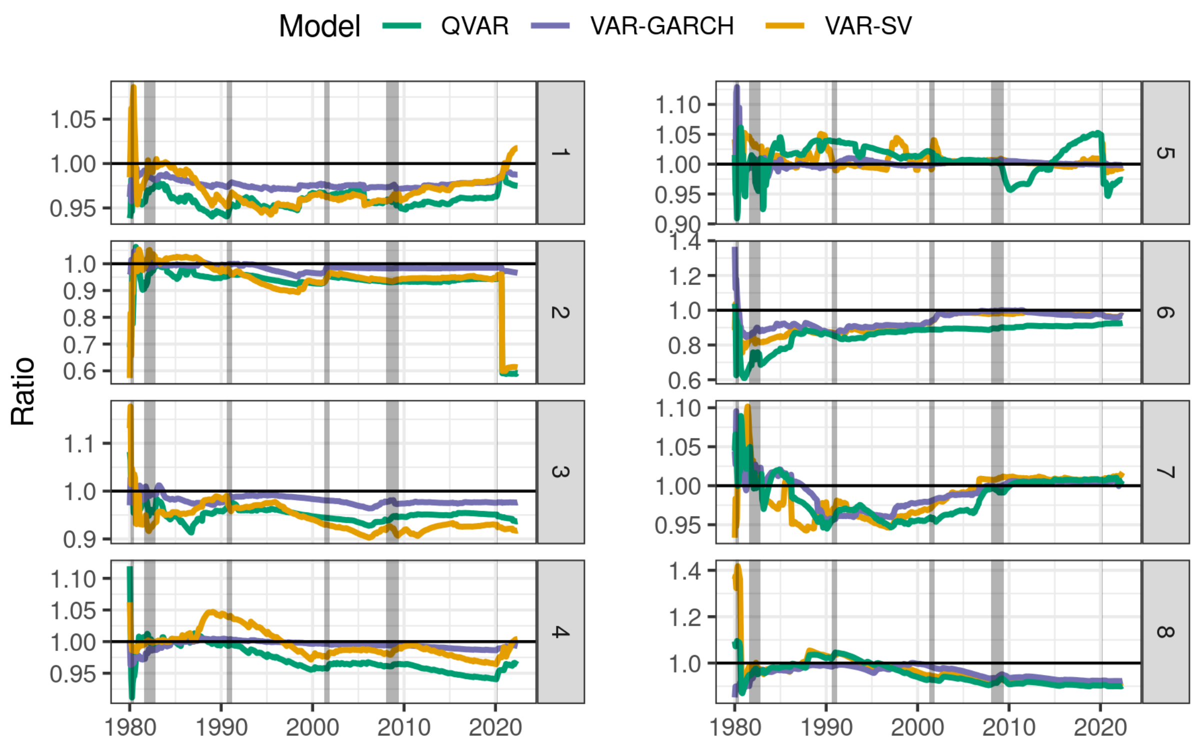


FIGURE A8. Recursive average ratios to VAR-N (Tails weighted CRPS, 6 months ahead)

The figure features the median of averages across variables in each group. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

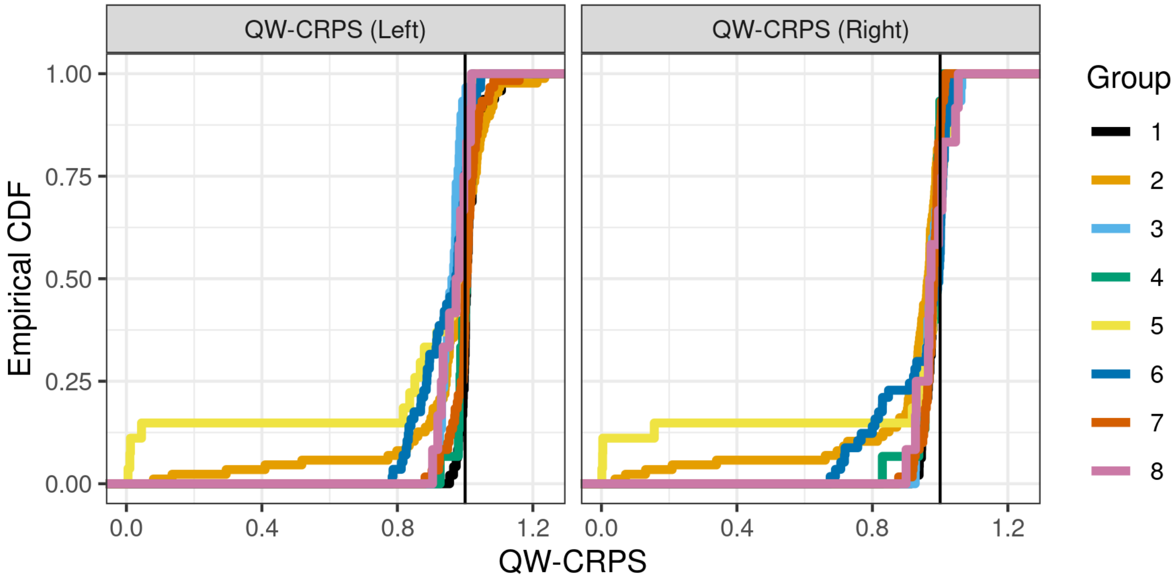


FIGURE A9. 3 Months ahead quantile weighted CRPS ratios (QVAR to VAR-N)

Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

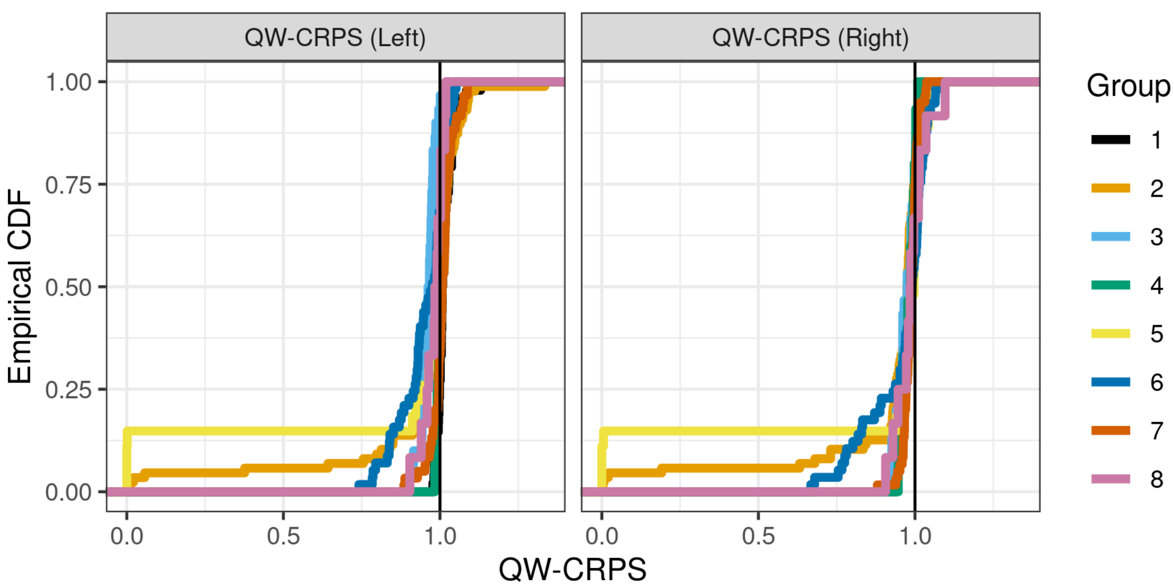


FIGURE A10. 6 Months ahead quantile weighted CRPS ratios (QVAR to VAR-N)

Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

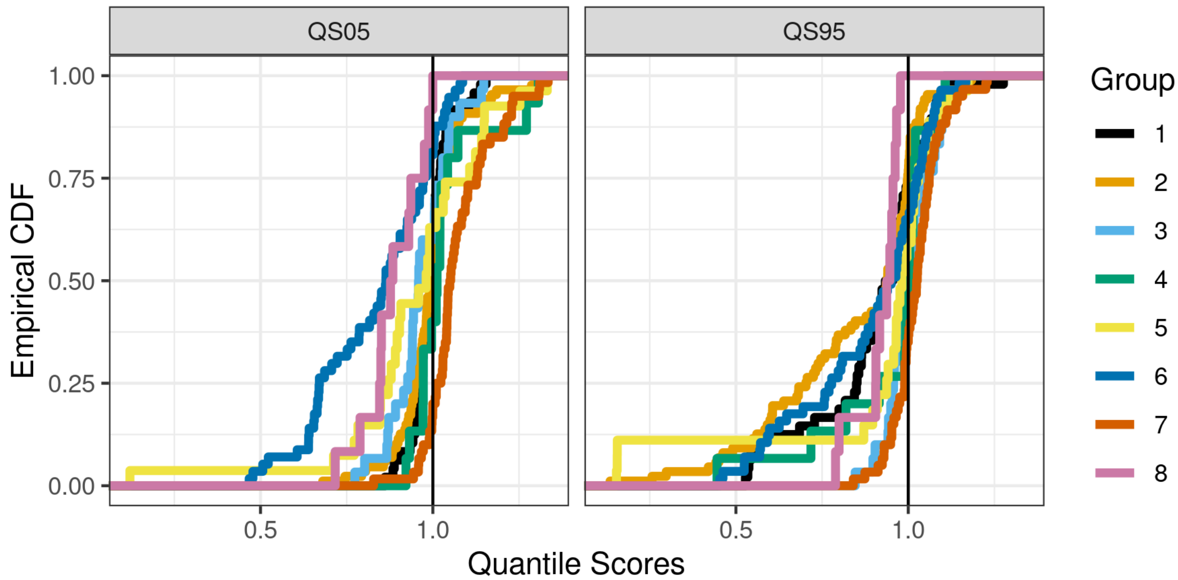


FIGURE A11. 1 Month ahead quantile score ratios (QVAR to VAR-N)

Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

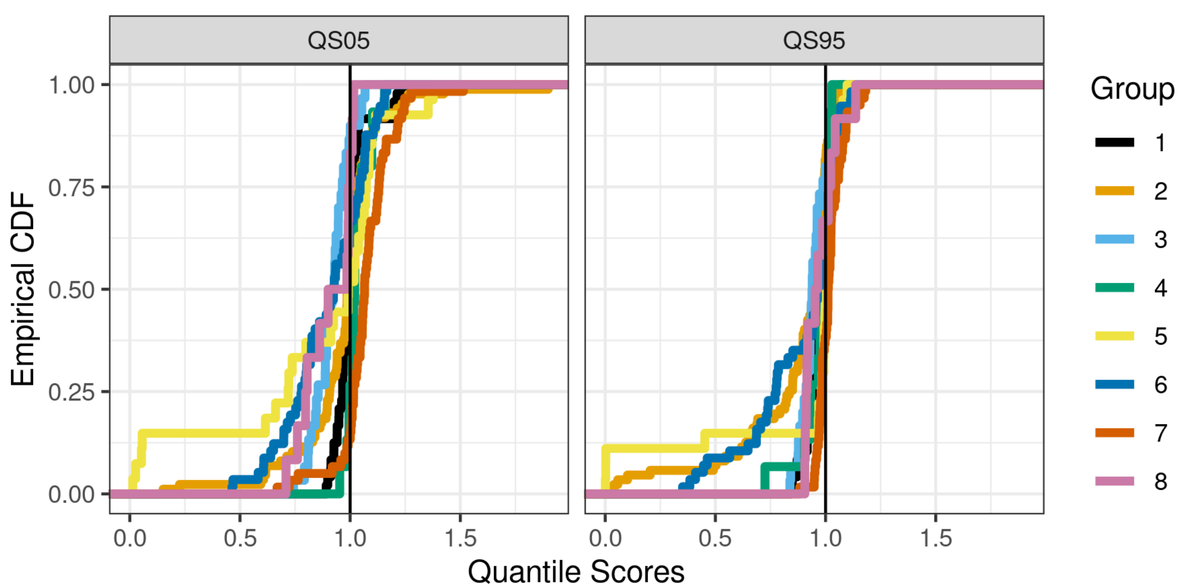


FIGURE A12. 3 Months ahead quantile score ratios (QVAR to VAR-N)

Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

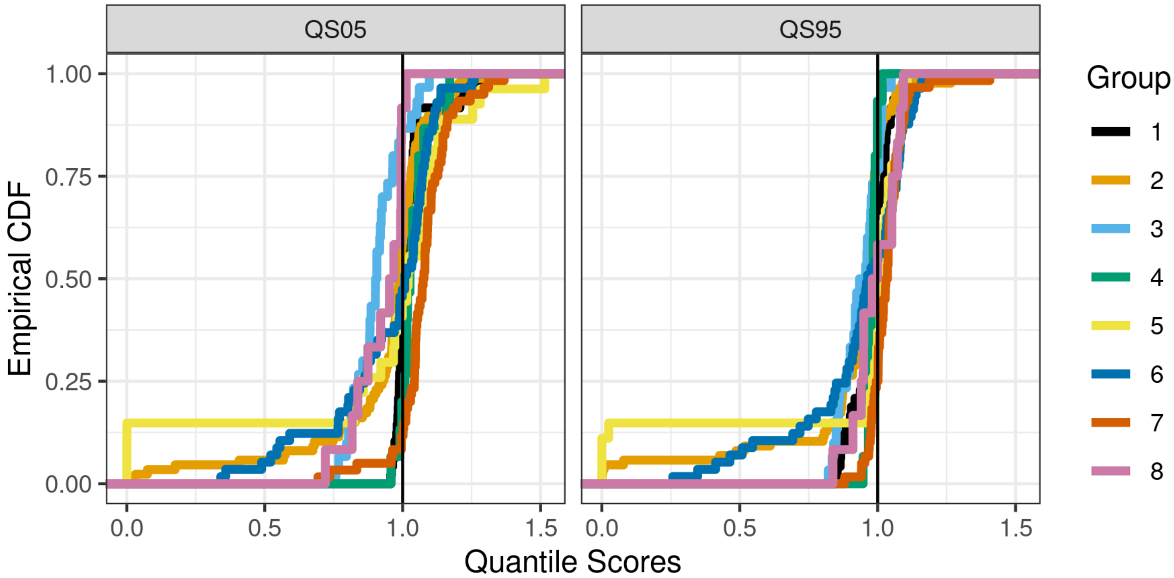


FIGURE A13. 6 Months ahead quantile score ratios (QVAR to VAR-N)

Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

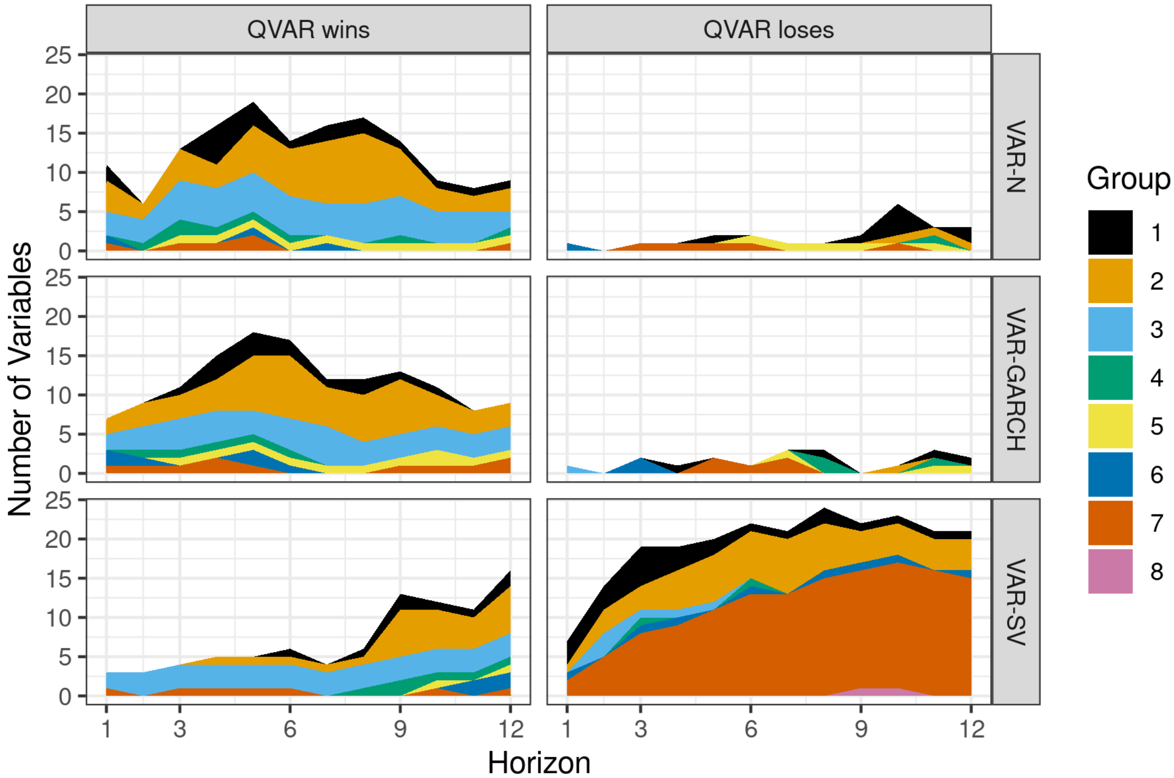
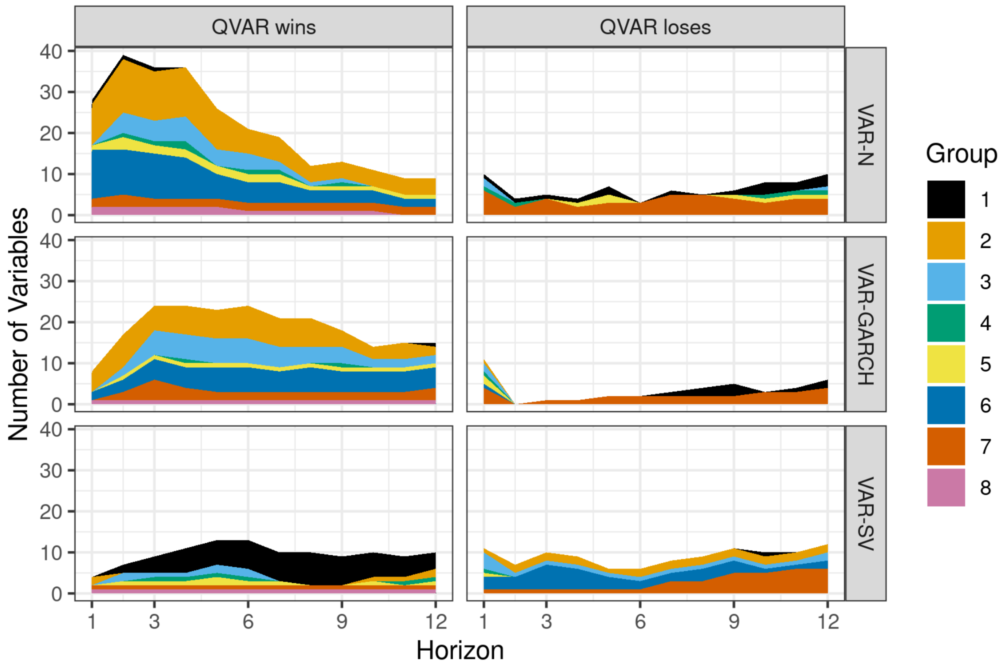
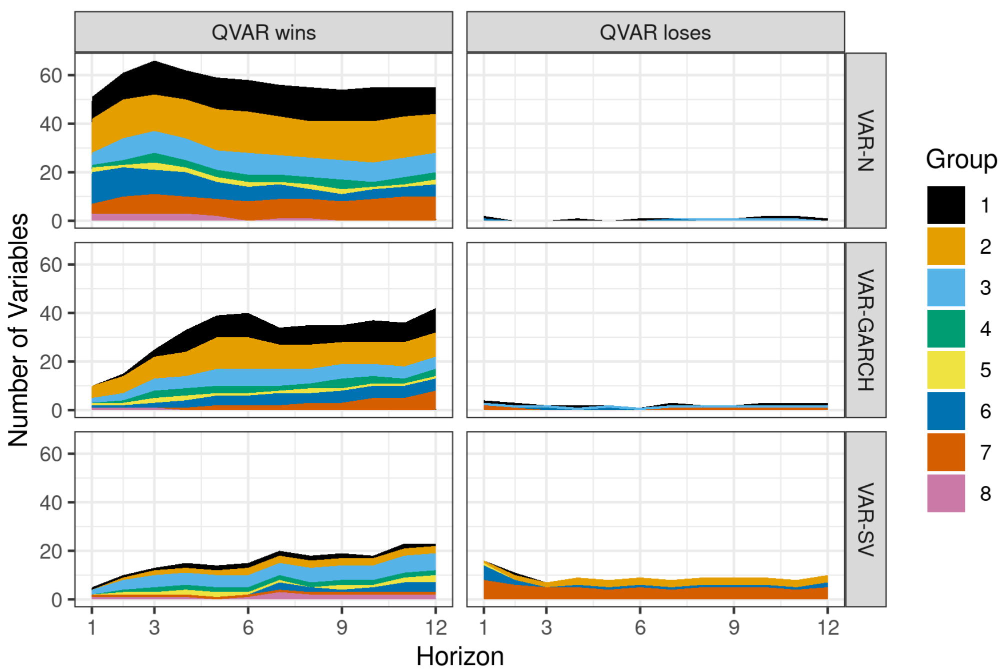


FIGURE A14. QVAR and benchmark model comparison (Square loss)

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).



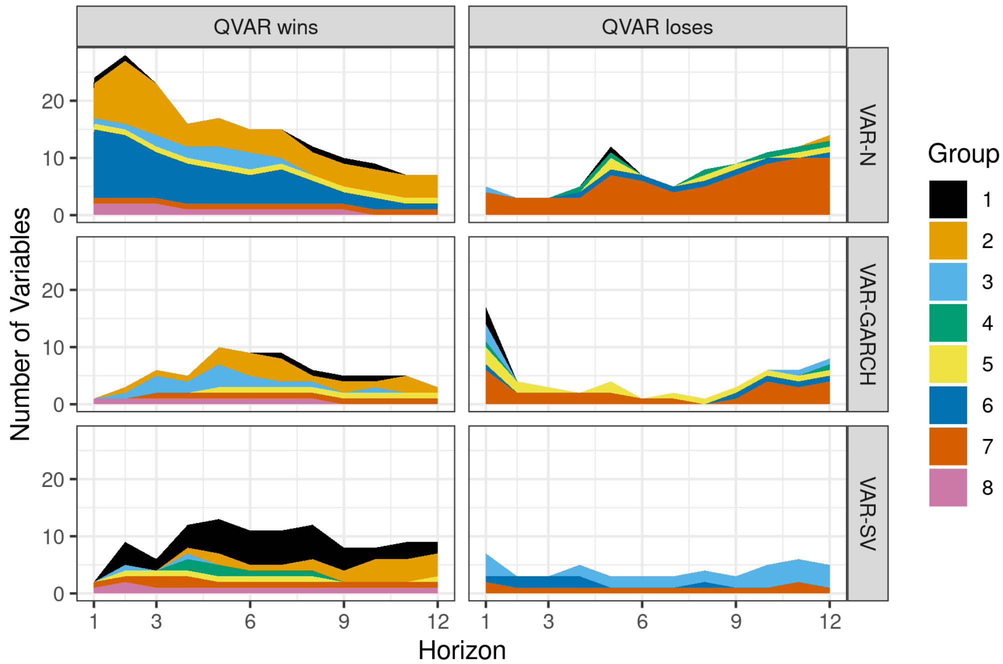
Left weighted CRPS



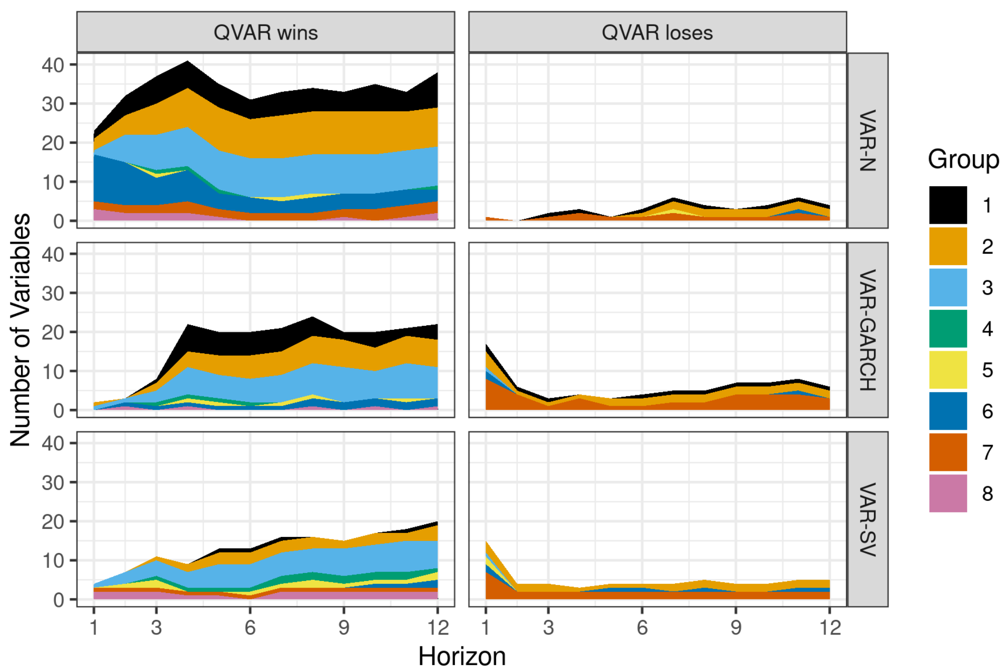
Right weighted CRPS

FIGURE A15. QVAR and benchmark model comparison in each tail

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).



5th quantile score



95th quantile score

FIGURE A16. QVAR and benchmark model comparison in each tail (Quantile score)

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

Appendix C. QFAVAR Results

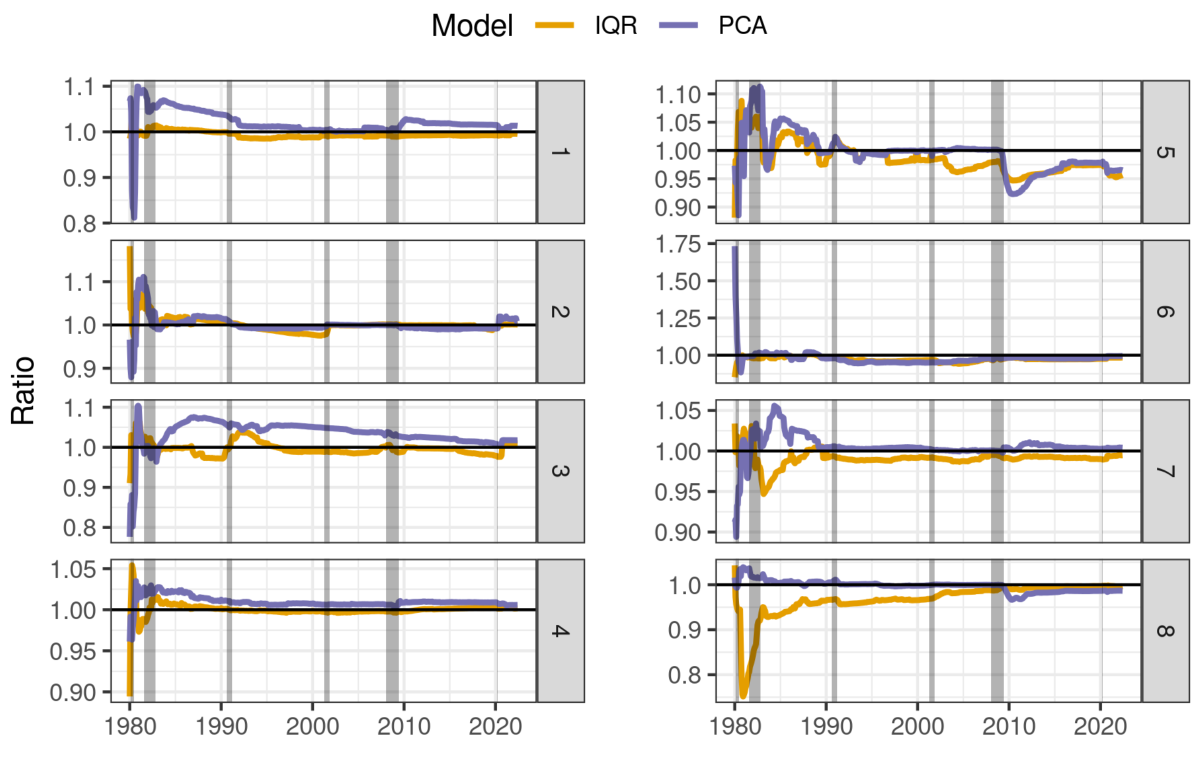
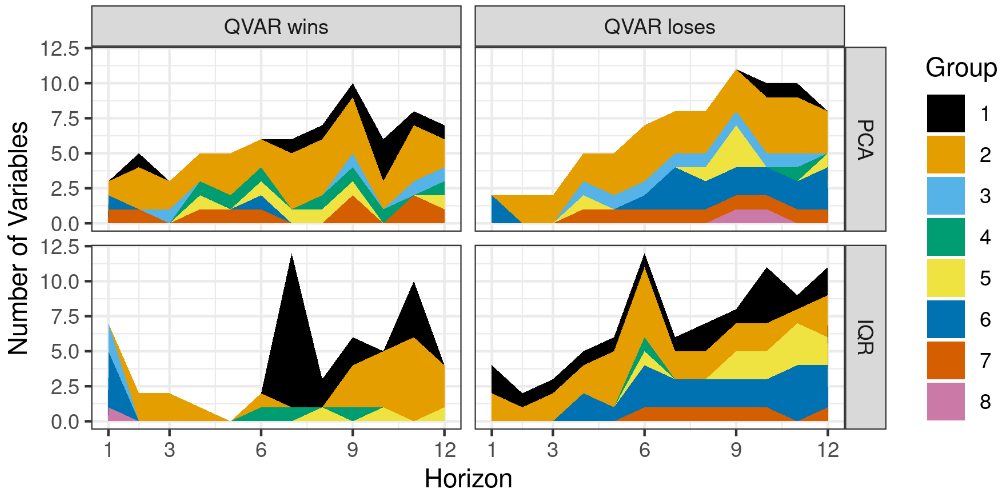
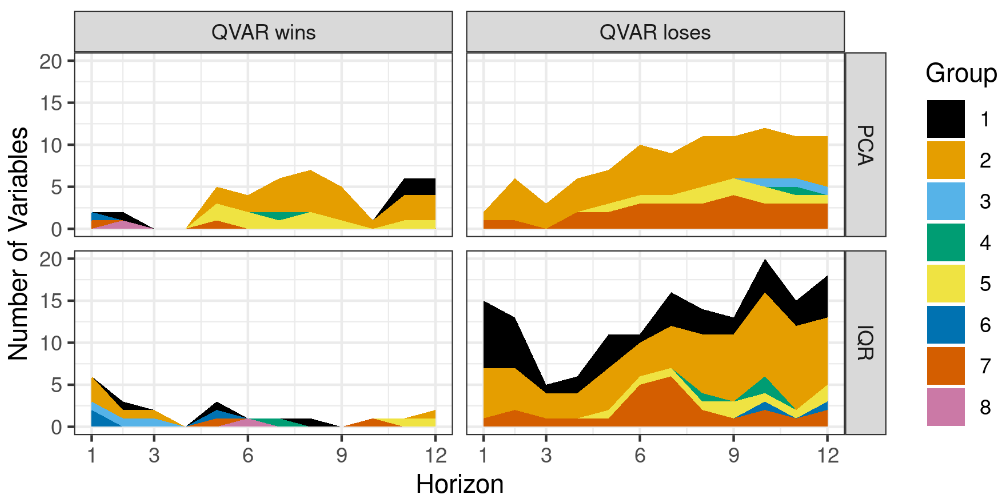


FIGURE A17. Recursive average ratios to QVAR (Tails weighted CRPS, 6 months ahead)

The figure features the median of averages across variables in each group. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).



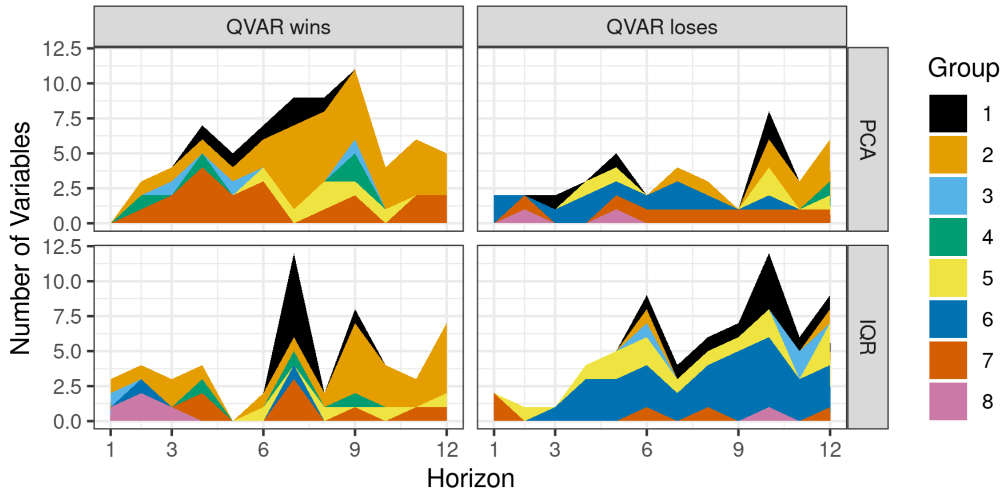
Left weighted CRPS



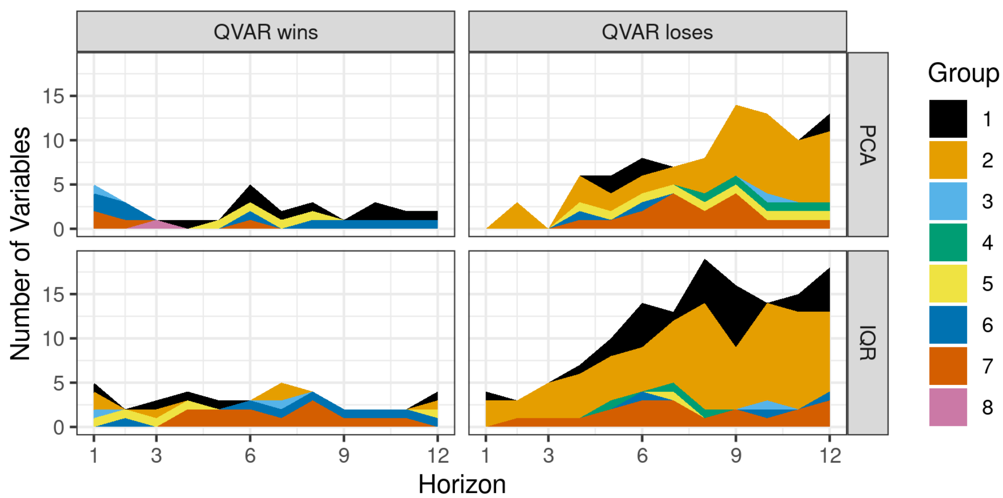
Right weighted CRPS

FIGURE A18. QFAVAR and QVAR comparison in each tail

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).



5th quantile score



95th quantile score

FIGURE A19. QFAVAR and QVAR comparison in each tail (Quantile scores)

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

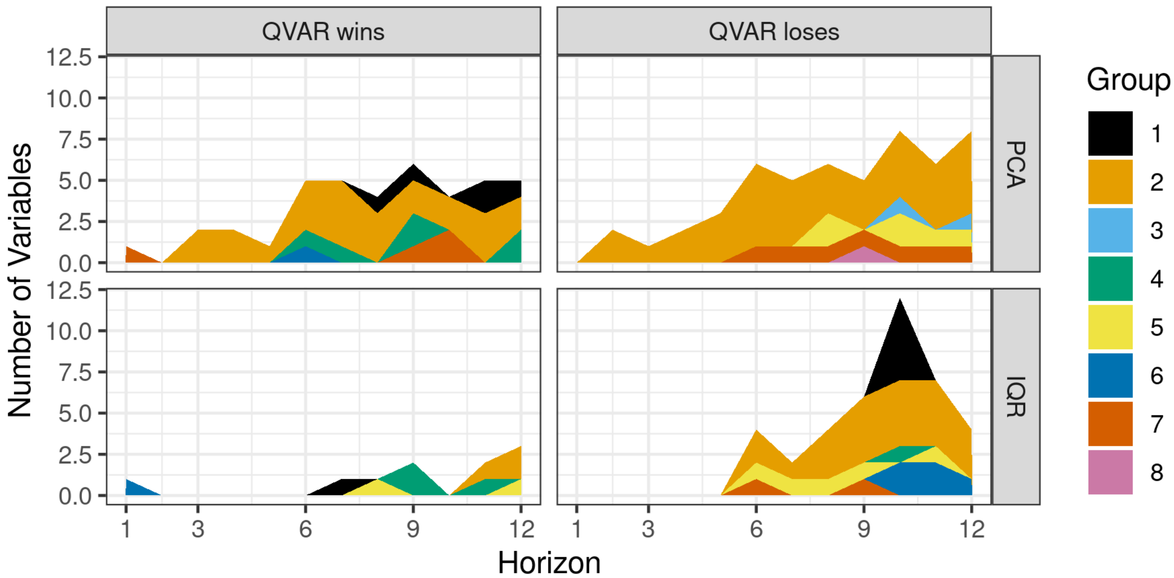


FIGURE A20. QFAVAR and QVAR comparison (Square loss)

Winning or losing requires 5% significance in the Diebold and Mariano (1995) test. Groups: (1) Output and income (16 variables), (2) Labor market (29 variables), (3) Housing (10 variables), (4) Consumption, orders and inventories (5 variables), (5) Money and credit (9 variables), (6) Interest and exchange rate (19 variables), (7) Prices (20 variables) and (8) Stock market (4 variables).

Appendix D. Data Transformation

As in the reference documentation of FRED-MD, the transformation codes are (1) y_t, (2) Δy_t, (3) $\Delta^2 y_t$, (4) $ln y_t$, (5) $\Delta ln y_t$, (6) $\Delta^2 ln y_t$ and (7) $y_t / y_{t-1} - 1$.

TABLE A3. Data Transformation

ID	Description	Used	FRED
RPI	Real Personal Income	5	5
W875RX1	Real personal income ex transfer receipts	5	5
INDPRO	IP Index	5	5
IPFPNSS	IP: Final Products and Nonindustrial Supplies	5	5
IPFINAL	IP: Final Products (Market Group)	5	5
IPCONGD	IP: Consumer Goods	5	5
IPDCONGD	IP: Durable Consumer Goods	5	5
IPNCONGD	IP: Nondurable Consumer Goods	5	5
IPBUSEQ	IP: Business Equipment	5	5
IPMAT	IP: Materials	5	5

Table A3. Data Transformation (Continued)

ID	Description	Used	FRED
IPDMAT	IP: Durable Materials	5	5
IPNMAT	IP: Nondurable Materials	5	5
IPMANSICS	IP: Manufacturing (SIC)	5	5
IPB51222s	IP: Residential Utilities	5	5
IPFUELS	IP: Fuels	5	5
CUMFNS	Capacity Utilization: Manufacturing	1	2
HWI	Help-Wanted Index for United States	5	2
HWIURATIO	Ratio of Help Wanted/No. Unemployed	4	2
CLF16OV	Civilian Labor Force	5	5
CE16OV	Civilian Employment	5	5
UNRATE	Civilian Unemployment Rate	1	2
UEMPMEAN	Average Duration of Unemployment (Weeks)	1	2
UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	1	5
UEMP5TO14	Civilians Unemployed for 5-14 Weeks	1	5
UEMP15OV	Civilians Unemployed - 15 Weeks & Over	1	5
UEMP15T26	Civilians Unemployed for 15-26 Weeks	1	5
UEMP27OV	Civilians Unemployed for 27 Weeks and Over	1	5
CLAIMSx	Initial Claims	5	5
PAYEMS	All Employees: Total nonfarm	5	5
USGOOD	All Employees: Goods-Producing Industries	5	5
CES1021000001	All Employees: Mining and Logging: Mining	5	5
USCONS	All Employees: Construction	5	5
MANEMP	All Employees: Manufacturing	5	5
DMANEMP	All Employees: Durable goods	5	5
NDMANEMP	All Employees: Nondurable goods	5	5
SRVPRD	All Employees: Service-Providing Industries	5	5
USTPU	All Employees: Trade, Transportation & Utilities	5	5
USWTRADE	All Employees: Wholesale Trade	5	5
USTRADE	All Employees: Retail Trade	5	5
USFIRE	All Employees: Financial Activities	5	5
USGOVT	All Employees: Government	5	5
CES0600000007	Avg Weekly Hours : Goods-Producing	1	1
AWOTMAN	Avg Weekly Overtime Hours : Manufacturing	1	2
AWHMAN	Avg Weekly Hours : Manufacturing	1	1
CES0600000008	Avg Hourly Earnings : Goods-Producing	5	6

Table A3. Data Transformation (Continued)

ID	Description	Used	FRED
CES2000000008	Avg Hourly Earnings : Construction	5	6
CES3000000008	Avg Hourly Earnings : Manufacturing	5	6
HOUST	Housing Starts: Total New Privately Owned	4	4
HOUSTNE	Housing Starts, Northeast	4	4
HOUSTMW	Housing Starts, Midwest	4	4
HOUSTS	Housing Starts, South	4	4
HOUSTW	Housing Starts, West	4	4
PERMIT	New Private Housing Permits (SAAR)	4	4
PERMITNE	New Private Housing Permits, Northeast (SAAR)	4	4
PERMITMW	New Private Housing Permits, Midwest (SAAR)	4	4
PERMITS	New Private Housing Permits, South (SAAR)	4	4
PERMITW	New Private Housing Permits, West (SAAR)	4	4
DPCERA3M086SBEA	Real personal consumption expenditures	5	5
CMRMTSPLx	Real Manu. and Trade Industries Sales	5	5
RETAILx	Retail and Food Services Sales	5	5
ACOGNO	New Orders for Consumer Goods	5	5
AMDMNOx	New Orders for Durable Goods	5	5
ANDENOx	New Orders for Nondefense Capital Goods	5	5
AMDMUOx	Unfilled Orders for Durable Goods	5	5
BUSINVx	Total Business Inventories	5	5
ISRATIOx	Total Business: Inventories to Sales Ratio	2	2
UMCSENTx	Consumer Sentiment Index	2	2
M1SL	M1 Money Stock	5	6
M2SL	M2 Money Stock	5	6
M2REAL	Real M2 Money Stock	5	5
BOGMBASE	Monetary Base	5	6
TOTRESNS	Total Reserves of Depository Institutions	5	6
NONBORRES	Reserves Of Depository Institutions	7	7
BUSLOANS	Commercial and Industrial Loans	5	6
REALLN	Real Estate Loans at All Commercial Banks	5	6
NONREVSL	Total Nonrevolving Credit	5	6
CONSPI	Nonrevolving consumer credit to Personal Income	5	2
DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding	5	6
DTCTHFNM	Total Consumer Loans and Leases Outstanding	5	6
INVEST	Securities in Bank Credit at All Commercial Banks	5	6

Table A3. Data Transformation (Continued)

ID	Description	Used	FRED
FEDFUNDS	Effective Federal Funds Rate	1	2
CP3Mx	3-Month AA Financial Commercial Paper Rate	1	2
TB3MS	3-Month Treasury Bill:	1	2
TB6MS	6-Month Treasury Bill:	1	2
GS1	1-Year Treasury Rate	1	2
GS5	5-Year Treasury Rate	1	2
GS10	10-Year Treasury Rate	1	2
AAA	Moody's Seasoned Aaa Corporate Bond Yield	1	2
BAA	Moody's Seasoned Baa Corporate Bond Yield	1	2
COMPAPFFx	3-Month Commercial Paper Minus FEDFUNDS	1	1
TB3SMFFM	3-Month Treasury C Minus FEDFUNDS	1	1
TB6SMFFM	6-Month Treasury C Minus FEDFUNDS	1	1
T1YFFM	1-Year Treasury C Minus FEDFUNDS	1	1
T5YFFM	5-Year Treasury C Minus FEDFUNDS	1	1
T10YFFM	10-Year Treasury C Minus FEDFUNDS	1	1
AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS	1	1
BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS	1	1
TWEXAFEGSMTHx	Trade Weighted U.S. Dollar Index	5	5
EXSZUSx	Switzerland / U.S. Foreign Exchange Rate	5	5
EXJPUSx	Japan / U.S. Foreign Exchange Rate	5	5
EXUSUKx	U.S. / U.K. Foreign Exchange Rate	5	5
EXCAUSx	Canada / U.S. Foreign Exchange Rate	5	5
WPSFD49207	PPI: Finished Goods	5	6
WPSFD49502	PPI: Finished Consumer Goods	5	6
WPSID61	PPI: Intermediate Materials	5	6
WPSID62	PPI: Crude Materials	5	6
OILPRICEx	Crude Oil, spliced WTI and Cushing	5	6
PPICMM	PPI: Metals and metal products:	5	6
CPIAUCSL	CPI : All Items	5	6
CPIAPPSL	CPI : Apparel	5	6
CPITRNSL	CPI : Transportation	5	6
CPIMEDSL	CPI : Medical Care	5	6
CUSR0000SAC	CPI : Commodities	5	6
CUSR0000SAD	CPI : Durables	5	6
CUSR0000SAS	CPI : Services	5	6

Table A3. Data Transformation (Continued)

ID	Description	Used	FRED
CPIULFSL	CPI : All Items Less Food	5	6
CUSR0000SA0L2	CPI : All items less shelter	5	6
CUSR0000SA0L5	CPI : All items less medical care	5	6
PCEPI	Personal Cons. Expend.: Chain Index	5	6
DDURRG3M086SBEA	Personal Cons. Exp: Durable goods	5	6
DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods	5	6
DSERRG3M086SBEA	Personal Cons. Exp: Services	5	6
S&P 500	S&P's Common Stock Price Index: Composite	5	5
S&P: indust	S&P's Common Stock Price Index: Industrials	5	5
S&P div yield	S&P's Composite Common Stock: Dividend Yield	1	2
S&P PE ratio	S&P's Composite Common Stock: Price-Earnings Ratio	1	5
VIXCLSx	VIX	1	1